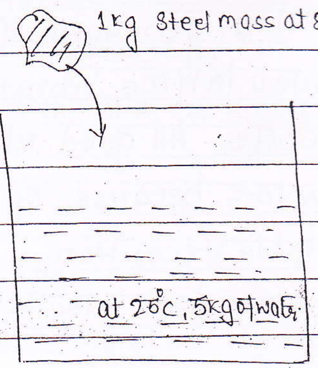
HMT

Samar Gaurabh

PAGE No.
DATE

③ # CONDUCTION




By Thermodynamics

Heat lost by steel body = Heat gained by water

(calorimetric eqn) at eqlbm

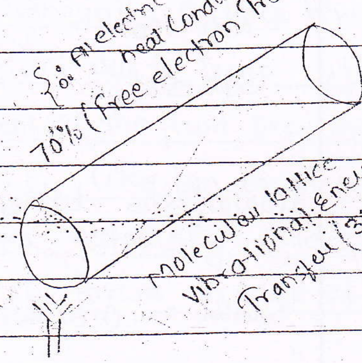
$M_{st} \times Cp_{steel} \; (85 - T_{final}) = M_w \times Cp_w \times (T_{final} - 25)$

By heat transfer we can say about the time taken to attain that temp. and rate of heat transfer.

## Difference b/w Thermodynamics & Heat transfer

In thermodynamics we deals with system in equilibrium i.e. we calculate how much heat energy is reqd to bring a system from one eqlbm state to another but in heat transfer we deal with at what rate heat energy gets transferred from high temp. body to low temperature body in Watts.

## Modes of Heat Transfer




Conduction :- (due to perfect lattice structure)

Diamond (non-metal) → K = 2200 W/mK

D. order ↓

Silver – K = 405 W/mK

Copper – K = 385 W/mK

Aluminium – K = 200 W/mK

Steels – K = 17 to 35 W/mK

Generally based on lattice arrangements Solids are classified as

→ Crystalline (Diamond, Quartz) — Perfect lattice structure

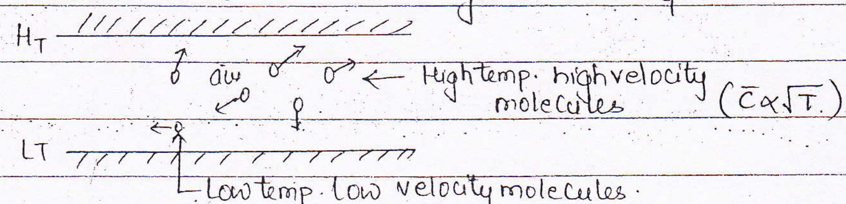
→ Amorphous (Glass) – Voids b/w the atoms so energy is little difficult to be transferred.

Conduction is the mode of heat transfer which generally occurs in solids due to temp. difference by molecular lattice vibrational energy and also by free electron transfer. All good electrical conductors are also good heat conductors because of presence of free electrons, Diamond is a notable exception.

Ex- All metals, as they have abundant free electrons.

We always use fins only with air.

**Note**:- Conduction also occurs in gases and liquids.




$K_{air} = 0.024$ W/mK (Conduction is due to molecular momentum ~~Energy~~ Transfer)

Insulators (solid)

Glass Wool $K = 0.075$ W/mK

Refractory brick

~~Abse~~ Asbestos $K = 0.2$ W/mK

Rock wool

Saw dust

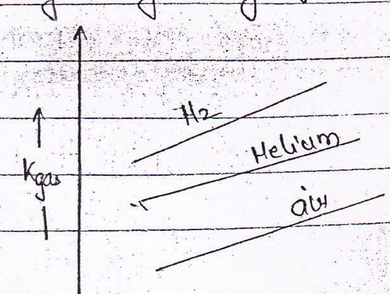
Liquids are better conductor than air.

$K_{water} = 0.62$ W/mK

$K_{Hg} = 8.1$ W/mK (liquid metal as its conductivity near to steel)

least vapour pressure and nearly every thing floats in mercury.

Conductivity of gases changes with temp.



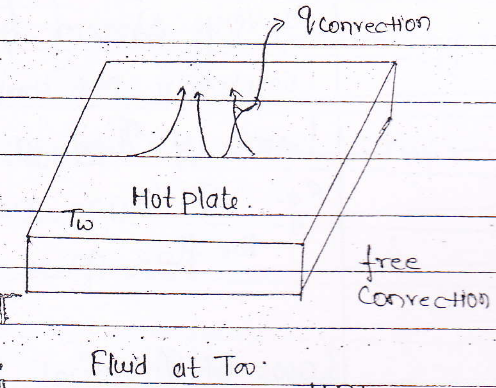
Thermal Conductivity (k) is a thermo-physical property of a material.
Variation of 'k' for solids is unpredictable.

## Convection:-

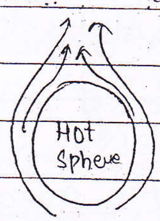
In heat transfer we assume isobaric process.

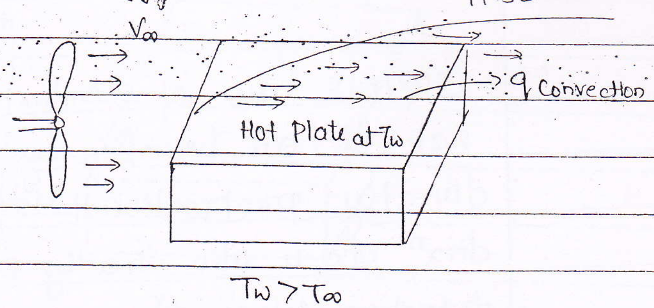
$$\frac{p}{\rho\downarrow} = RT\uparrow$$

density difference leads to flow of fluid "up" taking up the thermal energy.




Water cools 25 times faster than air.

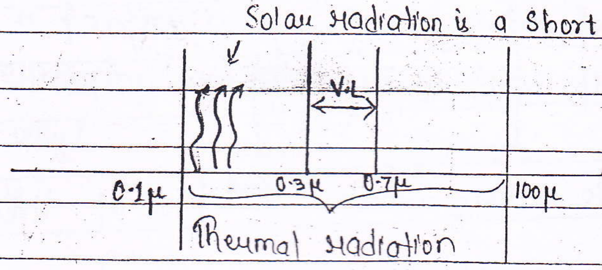



Convection is the mode of heat transfer which generally occurs b/w a hot solid body and the surrounding colder fluid due to temp. difference associated with macroscopic bulk displacement of the fluid over the solid which is provided by an external agency like fan or blower in the case of forced convection or which occurs naturally due to density changes and the resulting buoyancy forces in case of free convection.

## Radiation:-

Solar radiation is a short wavelength radiation.



Vaccum — ether



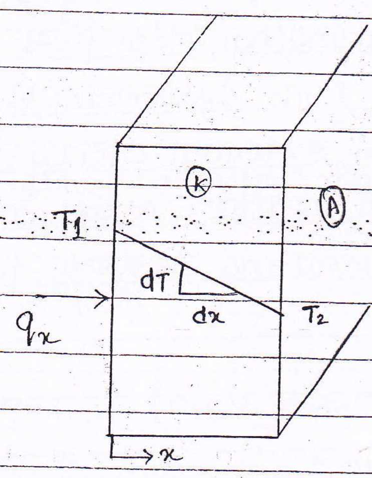
Radiation is a mode of heat transfer which doesnot require any material medium for its transfer, hence occurs by electromagnetic wave propagation travelling with the speed of light.
All bodies at all temp. emit thermal radiation except body at 0K.
Radiation is a surface phenomenon. The rate of emission being a very strong $f^n$ of absolute temp. of body. Radiation Completely Predominates over Conduction and Convection at high temp. diff.

~~Laws of Thermal Radiation~~

Eg- In large power boiler the mode of heat transfer b/w the flue gases & refractory wall is predominantly by radiation.

## Laws of Thermal Radiation

0. Fourier's Law of Conduction :- The law states that the rate of heat transfer by Conduction in a given $dir^n$ is directly proportional to the temp. gradient along that $dir^n$ and is also directly proportional to the area of heat transfer lying perpendicular to the $dir^n$ of heat transfer.




$$\boxed{q_x \propto -\left(\frac{dT}{dx}\right)}$$

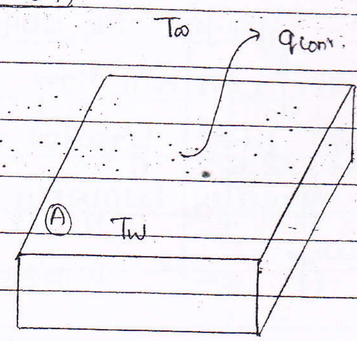
-ve Sign to satisfy Clausius $II^{nd}$ law of thermodynamics.

$$\Rightarrow \boxed{q_x = -KA\left(\frac{dT}{dx}\right)} \text{ Watts.}$$

'K' tells about the ability of the material to allow the heat energy to get conducted through it. ( W/m°c or W/mK same ).
Refrigerator wall are made from polyurethene foam ($\sim 10^{-6}$ W/mK) (PUF)

## Newton's law of Cooling (for Convection)

The law states that the rate of heat transfer b/w a hot solid body and the surrounding cold fluid due to convection is directly proportional to the temp. difference b/w them and is also directly proportional to the area of contact b/w them.




A = Surface area of body exposed to the fluid.

$$q_{conv} \propto (T_w - T_\infty)$$
$$\propto A$$

$$\Rightarrow q_{conv} = h A (T_w - T_\infty)$$
$$= h A \Delta T \text{ Watts.}$$

$h \rightarrow$ Convective heat transfer Coefficient in W/m²K ( Film H.T Coefficient).

Unlike k, h is not a property of a material but depends upon some of the thermophysical properties of the fluid like $\rho, C_p, \mu, k$ (of fluid only)
Absolute viscosity

In forced Convection heat transfer 'h' is a fxn of

$h = f(\vec{V}, D, \rho, \mu, C_p, K)$

D ↑ Characteristic dimension.

In free Convection heat transfer $h = f(g, \beta, \Delta T, L, \mu, \rho, C_p, K)$

$\beta \rightarrow$ Isobaric volume expansion coefficient of fluid.

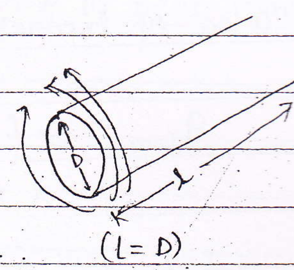
$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$$

it takes into account the buoyancy effect.

$\beta$ signifies the ability of the fluid how much volume changes it will have for a given temp. change. Higher the value of $\beta$, greater the density changes of fluid resulting in greater buoyancy forces and easily the fluid moves away.

For air (Ideal gas) $\beta = \frac{1}{T}$ T is in kelvin

$L \rightarrow$ Characteristic length of body.




0 Stefan – Boltzmann Law

The law states that the radiation energy emitted from the surface of a black body per unit time is directly proportional to the fourth power of the absolute temp. of body

$E_b \propto T^4$ Where T is in kelvin

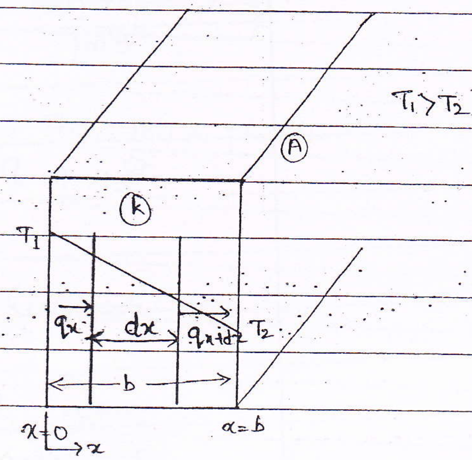
$$\boxed{E_b = \sigma T^4 \ W/m^2}$$

$\sigma$ - Stefan Boltzmann Constant $= 5.67 \times 10^{-8}\ W/m^2K^4$

A black body is the body which absorbs all the thermal radiation incident upon it. A thermally black body need not appear black in Colour to the human eye. Eg - Snow, Ice

## Integration of Fourier Law

01 Case I :-




Assumptions
- Steady state heat transfer
- One dimensional Conduction
  $T = f(x)$ only
- Uniform Thermal Conductivity
  $k = \text{Constant}$. ⇒ Temp. variation is linear.

by Fourier's Law

$$q_x = -kA\frac{dT}{dx}$$

Boundary Cond$^n$s

At $x=0 \Rightarrow T = T_1$

$x = b \Rightarrow T = T_2$

$$\Rightarrow \int_0^b q_x\,dx = \int_{T_1}^{T_2} -kA\,dT$$

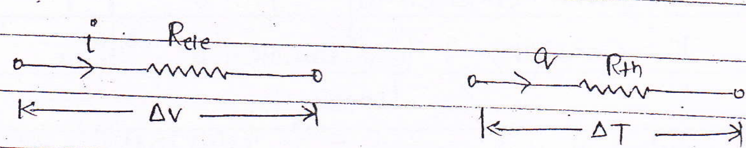
$q_x \neq f(x)$ i.e. $q_x = q_{x+dx}$ (to maintain steady state Cond$^n$s)

$$\Rightarrow q_x \times b = kA(T_1 - T_2)$$

$$\Rightarrow \boxed{q_x = \frac{kA(T_1 - T_2)}{b}} \text{ Watts}$$

## Electrical Analogy (similarity) of Heat Transfer

| Electrical | Thermal |
|---|---|
| i (Amps) → | q (watts) |
| ΔV (or) Emf → | ΔT °C |
| $R_{ele}$ in Ω → | $R_{th}$ |




$$R_{ele} = \frac{\Delta V}{i} \ \Omega \qquad R_{th} = \left(\frac{\Delta T}{q}\right) K/Watt$$

For slab

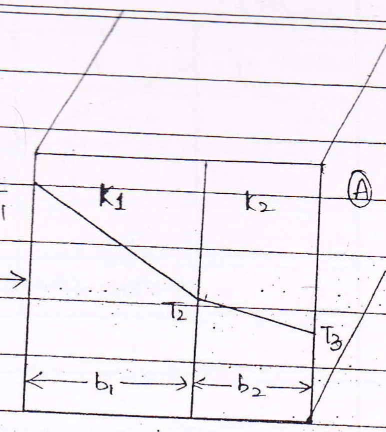
$$R_{th} = \frac{(T_1 - T_2)}{q} = \left(\frac{b}{KA}\right) K/Watt$$

Lesser the Conductivity of the Slab, higher will be the resistance, also Greater the thickness of Slab more the resistance.

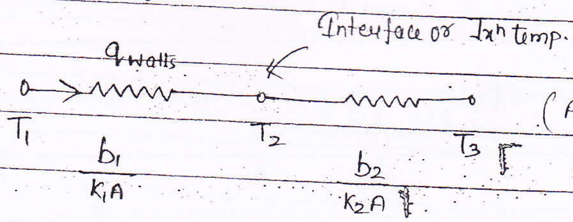
2. Case-II :- Conduction heat transfer from Composite Slab.

Assumptions

- Steady flow heat transfer
- One dimensional heat transfer
- Uniform 'k' values.




Thermal Circuit




(Always show this in problem)

q = Rate of heat transfer through Composite Slab

$$= \frac{T_1 - T_3}{\Sigma R_{th}} = \frac{(T_1 - T_3)}{\left(\frac{b_1}{k_1 A} + \frac{b_2}{k_2 A}\right)} \text{ Watts}$$

Heat flux = Heat transfer rate per unit area

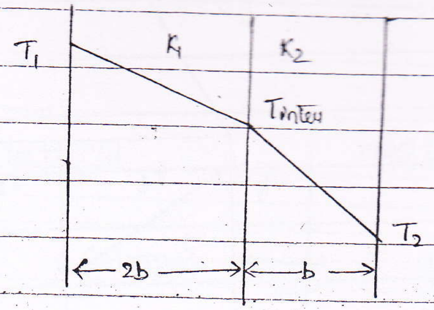
$$= \frac{q}{A} = \left(\frac{T_1 - T_3}{\frac{b_1}{k_1} + \frac{b_2}{k_2}}\right) \text{ W/m}^2$$

To get $T_2$

$$q = \frac{T_1 - T_2}{(b_1/kA)} \Rightarrow T_2 = \_\_\_\_ {}^\circ C.$$

$K_1 < k_2$ Since slope of temp gradient in 1 is greater than slope in 2

Q. In a Composite slab the temperature at the interface b/w two materials is equal to the average of the temperatures at the two ends. Assuming steady one-dimensional heat conduction, Which of the following statements is true about the respective thermal Conductivities.



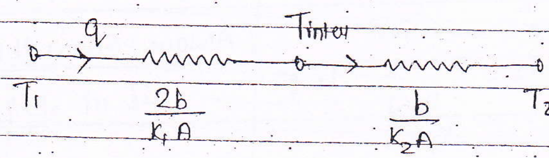

(a) $2k_1 = k_2$

(b) $k_1 = k_2$

(c). $2k_1 = 3k_2$

(d). $k_1 = 2k_2$

Ans.




$$T_{inter} = \frac{T_1 + T_2}{2}$$

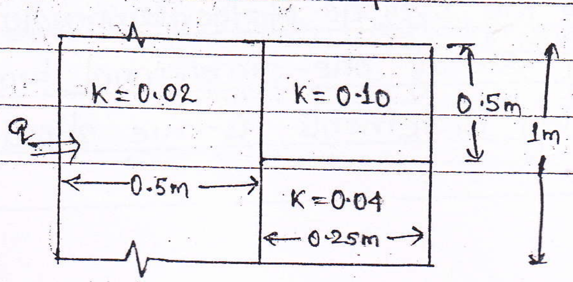
$$\Rightarrow q = \frac{T_1 - T_{inter}}{\frac{2\cancel{b}}{k_1 \cancel{A}}} = \frac{T_{inter} - T_2}{\frac{\cancel{b}}{k_2 \cancel{A}}}$$

$$\Rightarrow \frac{T_1 - \frac{(T_1 + T_2)}{2}}{\frac{2}{k_1}} = \frac{\frac{T_1 + T_2}{2} - T_2}{\frac{1}{k_2}}$$

$$\Rightarrow k_1 \left[\frac{T_1 - T_2}{4}\right] = \left(\frac{T_1 - T_2}{2}\right) k_2$$

$$\Rightarrow k_1 = 2k_2$$

Q. Heat flows through a Composite slab as shown below, the depth of the slab is 1m, the k values are in W/mK, the overall thermal resistance in K/W is



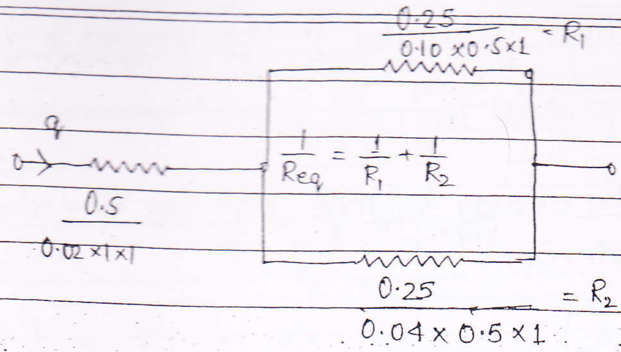

(a). 17.2

(b). 21.9

(c). 28.6

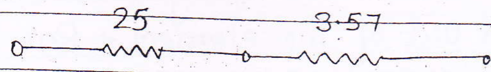
(d). 39.2

Ans.



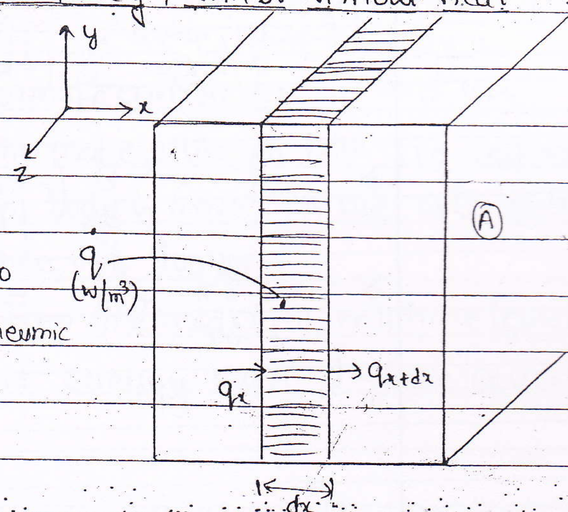

Depth is 1 to the Paper = 1m

$\therefore A = 1 \times 1\, m^2$




$\Rightarrow \quad R_{th} = 28.57 \ W/mK$

Generalised Conduction (Steady or Unsteady, With or Without heat generation, 3 dimensional) eqn :-




Let there be Uniform heat generation rate.

The heat generation may be due to passage of electric current or exothermic rxn or thermonuclear rxn (fission)

Consider a differential element of the slab of length 'dx'.

$$q_x \neq f(x,y,z) \quad \& \quad k = c$$

Let $q_x$ = Heat Conducted into element along x-dirn.

$$= -KA\left(\frac{dT}{dx}\right)$$

$q_{x+dx}$ = Heat Conducted Out of the element along x-dirn.

$$= q_x + \frac{\partial}{\partial x}(q_x)\,dx$$

Rate of heat generated in the element = $\dot{q} \times$ Volume of element
= $\dot{q} \times A \times dx$ Watts.

Writing the energy balance eqn for the x-dirxn conduction for the element.

Heat Conducted into the element + Heat generated in the element = Heat Conducted out of the element + Rate of change of internal energy w.r.t time.

$$\Rightarrow q_x + \dot{q}\, A dx = q_{x+dx} + \frac{\partial}{\partial \tau}(\text{I.E. of the element})$$

$$\Rightarrow \cancel{q_x} + \dot{q}\, A dx = \cancel{q_x} + \frac{\partial}{\partial x}(q_x)dx + \frac{\partial}{\partial \tau}(m c_p T)$$

$$\Rightarrow \dot{q}\cancel{A}\cancel{dx} = \frac{\partial}{\partial x}\left(-k\cancel{A}\frac{dT}{dx}\right)\cancel{dx} + \frac{\partial}{\partial \tau}\left(\rho \cancel{A}\cancel{dx}\, C_p T\right)$$

$$\Rightarrow \dot{q} = \frac{\partial}{\partial x}\left(-k\frac{dT}{dx}\right) + \frac{\partial}{\partial \tau}(\rho C_p T)$$

$$\Rightarrow k\frac{\partial^2 T}{\partial x^2} + \dot{q} = \rho C_p\left(\frac{\partial T}{\partial \tau}\right)$$ ← rate of change of temp. of element w.r.t time or rate of Cooling/heating.

Writing the energy balance eqn. ~~for~~ and the similarly for all the three Cartesian dirxns x, y, z simultaneously. We get,

$$k\frac{\partial^2 T}{\partial x^2} + k\frac{\partial^2 T}{\partial y^2} + k\frac{\partial^2 T}{\partial z^2} + \dot{q} = \rho C_p\left(\frac{\partial T}{\partial \tau}\right)$$

← not multiplied by 3 as Conduction taking place simultaneously in 3 dirxns.

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho C_p}{k}\left(\frac{\partial T}{\partial \tau}\right)$$

defining the property of material '$\alpha$' ~~as the~~ ratio b/w ~~Kinematic viscosi~~ Thermal Conductivity and Thermal heat capacity

$$\alpha = \left(\frac{k}{\rho \underbrace{C_p}_{J/kg\text{-}K}}\right) m^2/sec.$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\left(\frac{\partial T}{\partial \tau}\right)$$

$$\alpha_{gas} > \alpha_{liquid}$$

29/11/2011 Significance of $\alpha$ ($m^2/s$).

The ability of the material to allow the heat energy to get diffused or passed through the material. If '$\alpha$' is more, when the body is suddenly exposed to a changed thermal environment it takes less amount of time to come into thermal equilibrium with it i.e. response time is lesser. If the conductivity of body is more, the rate of conduction through the metal is faster hence $\alpha$ is higher.
If heat capacity of body will be lesser the material will have lesser storage ability of heat hence, the energy must diffuse quickly making the '$\alpha$' value higher.

Prandtl no. of a fluid (Dimensionless no.) $= \dfrac{\nu \leftarrow \text{Kinematic viscosity}}{\alpha \leftarrow \text{Thermal diffusivity}}$

$\Rightarrow$ K.V of a fluid $= \dfrac{\mu \leftarrow \text{Dynamic viscosity}}{\rho}$

$\Rightarrow$ $P_r = \dfrac{\mu C_p}{k}$ (a property of fluid)

It is the only property which is dimensionless

If conditions are steady,

then $\dfrac{\partial T}{\partial \tau} = 0$

If there is no heat generation,

$\dot{q} = 0$

Fourier Eq$^n$

$$\nabla^2 T = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\Rightarrow \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

Poisson's eq$^n$

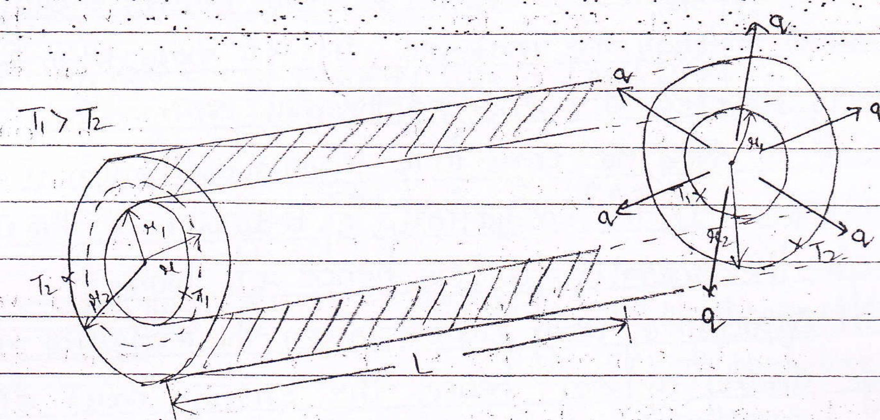
$$\nabla^2 T + \frac{\dot{q}}{k} = 0$$

$$\Rightarrow \nabla^2 T = 0$$

Laplace' equation in T.

Note:- Laplace' equation in T indicates 3-D Steady Conduction eq$^n$ without any heat generation.

Radial Conduction heat transfer through a hollow Cylinder




At $r = r_1 \Rightarrow T = T_1$

At $r = r_2 \Rightarrow T = T_2$

$T_1 > T_2$

Conduction heat transfer occurs radially outwards from inside surface at $T_1$ to outside surface at $T_2$.

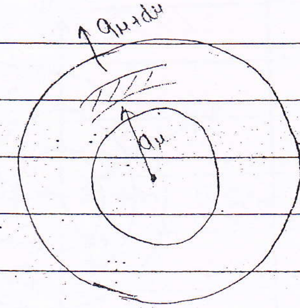
Note:- Unlike in case of slabs, here conduction heat transfer area keeps changing in the dir$^n$ of the heat flow.

At any radius $r$,

Area of Conduction Heat transfer $= 2\pi r L$

Assuming Steady state Radial Conduction heat transfer

By Fourier' Law of Conduction




$$q = -KA\frac{dT}{dr}$$

$$q = -K\,2\pi r L\frac{dT}{dr}$$

$$\Rightarrow \int_{r=r_1}^{r=r_2} q\frac{dr}{r} = \int_{T_1}^{T_2} -2\pi KL\,dT$$

$q_r = q_{r+dr}$

$\Rightarrow q \neq f(r)$

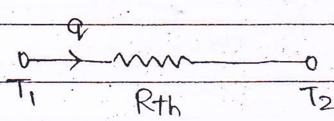
to satisfy steady state

$K$ = Constant

$$\Rightarrow q\ln\left(\frac{r_2}{r_1}\right) = 2\pi KL(T_1 - T_2)$$

$$\Rightarrow \boxed{q = \frac{2\pi KL(T_1 - T_2)}{\ln(r_2/r_1)}}$$

The equivalent thermal circuit is

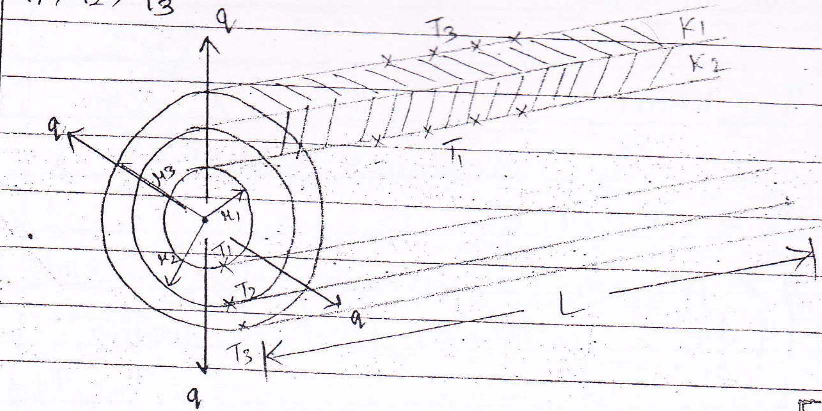



$$\Rightarrow R_{th} = \left(\frac{T_1 - T_2}{q}\right) = \frac{\ln(r_2/r_1)}{2\pi KL}\ \text{K/watt}$$

Note :- As $r_1 \to r_2$ i.e. as the thickness of the cylinder becoming smaller, Logarithmic fxn value also tends to zero which means resistance $\to 0$. Otherwise if K value of material is very high (a metal pipe) then also resistance will be lesser increasing the heat current.

## Radial Conduction Heat Transfer through a Composite ~~Transfer~~ Cylinder

$T_1 > T_2 > T_3$




$T = f(r)$

Assume steady, One dimensional Radial Conduction through Composite Cylinder.

**Thermal Circuit**

$$T_1 \xrightarrow{q} \underset{\frac{\ln(r_2/r_1)}{2\pi K_1 L}}{\text{WWW}} \; T_2 \; \underset{\frac{\ln(r_3/r_2)}{2\pi K_2 L}}{\text{WWW}} \; T_3$$

$\therefore$ $q$ = Rate of Heat Transfer $= \dfrac{T_1 - T_3}{\left(\dfrac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi K_1 L} + \dfrac{\ln\left(r_3/r_2\right)}{2\pi K_2 L}\right)}$

**Q.** A stainless steel tube ($K_s = 19$ W/mK) of 2 cm inner diameter and 5 cm outer diameter is insulated with 3 cm thick asbestos ($K_a = 0.2$ W/mK). If the temp. difference b/w the innermost and outermost surfaces is 600°c, The heat transfer rate per unit length is

(a) 0.94 W/m (b) 9.44 W/m (c) 944.72 W/m (d). 9447.21 W/m

**Ans** $T_1 - T_3 = \Delta T = 600^\circ C$

$$R_{th} = \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_1 L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_2 L}$$

$r_1 = 1\,cm$, $r_2 = 2.5\,cm$, $r_3 = 2.5 \overset{+3}{\cancel{+1.5}} = \cancel{4\,cm}\ 5.5\,cm$

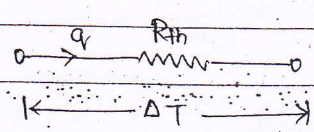
$$\Rightarrow q = \frac{600}{\frac{\ln\left(\frac{2.5}{1}\right)}{2\pi \times 19 \times 1} + \frac{\ln\left(\frac{5.5}{2.5}\right)}{2\pi \times 0.2 \times 1}}$$

$$= \underline{944.72\ W/m}\ \text{Ans}$$

$$\boxed{W/m^\circ C = W/mK}$$

## Convection Thermal resistance

$$q_{conv.} = hA\Delta T$$



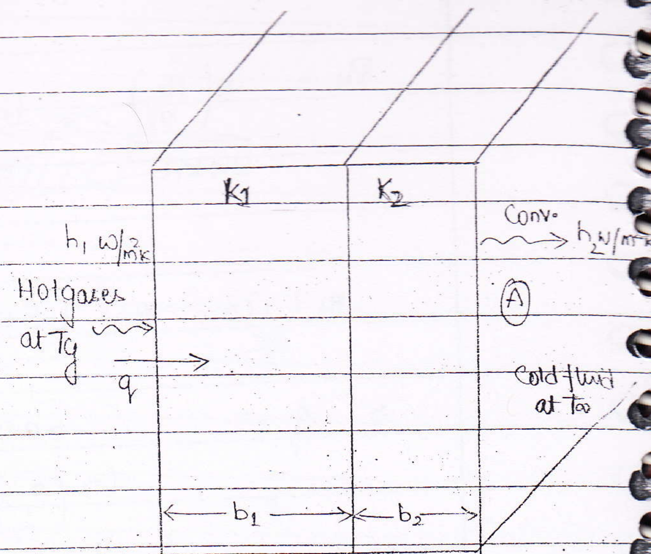

$$\Rightarrow R_{th} = \frac{\Delta T}{q} = \frac{1}{hA}\ K/Watts$$

$A =$ Area of Contact.

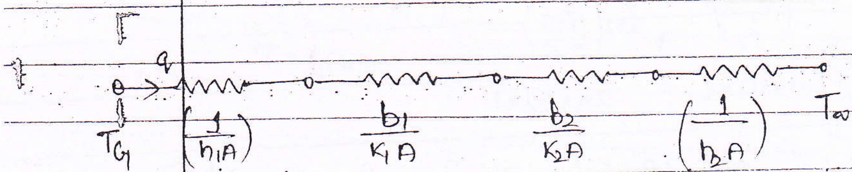
higher the value of Convective ~~Coefficient~~ Coefficient, lower the thermal resistance of Convection.

## Conduction, Convection Heat Transfer from a Composite slab

Assume Steady state one dimensional H.T. through Composite slab.




Thermal Circuit




$h_1$ and $h_2$ even though of same fluid may not be same as they depend on various parameters.

$h_1 \rightarrow$ Convective heat transfer Coefficient at gas side.

$h_2 \rightarrow$ Convective heat transfer Coefficient on Cold side.

$$\therefore \quad q = \frac{T_G - T_\infty}{\left(\frac{1}{h_1A} + \frac{b_1}{k_1A} + \frac{b_2}{k_2A} + \frac{1}{h_2A}\right)}$$

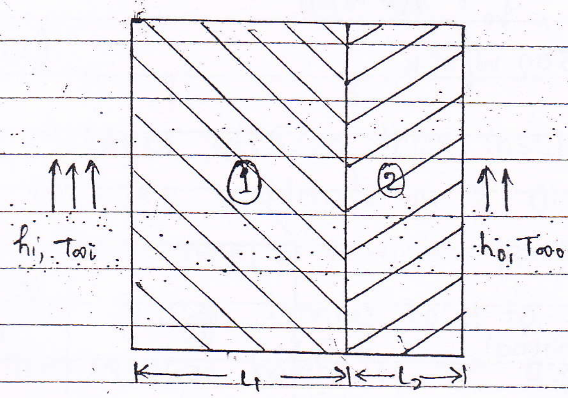
$$\Rightarrow \frac{q}{A}\ (\text{Heat flux}) = \frac{T_G - T_\infty}{\left(\frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{1}{h_2}\right)}\ W/m^2 \quad \text{———— (i)}$$

**Overall Heat Transfer Coefficient (U) $W/m^2K$**

It is the parameter which takes into account all the modes of heat transfer into a single entity and hence is defined from the equation $q = UA\Delta T$ ———— (ii)

$$\Rightarrow \boxed{\frac{1}{U} = \frac{1}{h_1} + \frac{b_1}{k_1} + \frac{b_2}{k_2} + \frac{1}{h_2}}$$

Gate-2009 Consider steady state heat conduction across the thickness in a plane composite wall shown in fig. exposed to convection conditions on both sides.




Assuming negligible contact resistance b/w the wall surfaces, the interface temp. T in °c of the two walls will be

$L_1 = 0.30m$, $h_i = 20 W/m^2K$, $T_{\infty i} = 20°c$, $K_1 = 20 W/mK$

$L_2 = 0.15m$, $h_o = 50 W/m^2K$, $T_{\infty o} = -2°c$, $K_2 = 50 W/mK$

Ans.
$$\frac{q}{A} = \frac{T_{\infty i} - T_{\infty o}}{\left(\frac{1}{h_i} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{1}{h_o}\right)}$$

$$= \frac{20 + 2}{\left(\frac{1}{20} + \frac{0.3}{20 \times 1} + \frac{0.15}{50} + \frac{1}{50}\right)} = 250 W/m^2$$

$$\Rightarrow 250 = \frac{T_2 - T_{\infty o}}{\left(\frac{0.15}{50} + \frac{1}{50}\right)} = \frac{T_2 + 2}{\left(\frac{0.15}{50} + \frac{1}{50}\right)}$$

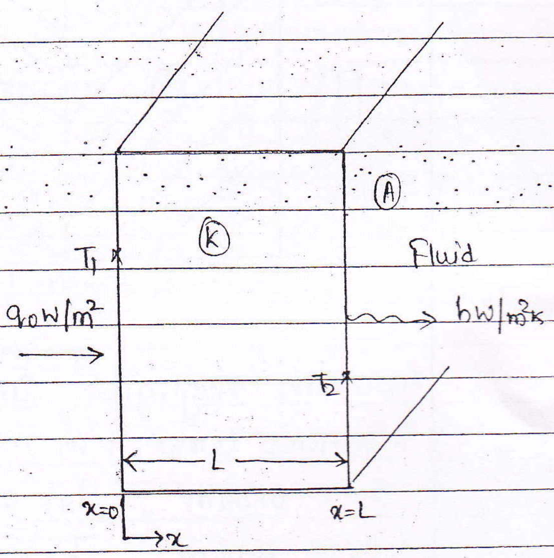
$$\Rightarrow T_2 = 3.75°C \text{ Ans.}$$

IES-2003 An Iron plate of thickness 'L' and thermal Conductivity 'k' is Subjected to a Constant heat flux $q_0$, $W/m^2$ ~~an~~ at the boundary Surface at $x=0$. from the other boundary Surface at $x=L$ the heat is dissipated by Convection into a fluid at temp $T_\infty$ With a heat transfer Coefficient 'h'. Develop the expressions for the surface temp. $T_1$ & $T_2$ at the surfaces $x=0$ and $x=L$ resp. for the following data Calculate the Surface temp. $T_1$ and $T_2$ if

$L = 2\,cm$, $k = 20\,W/m^\circ C$, $q_0 = 10^5\,W/m^2$

$T_\infty = 50^\circ C$ & $h = 500\,W/m^2K$ (15 marks)

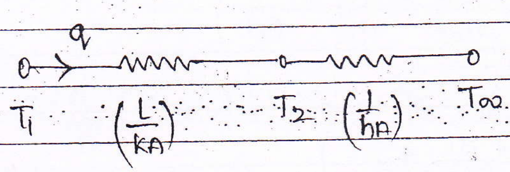
Ans.




Assumption

- Steady state One dimensional Conduction - Convection Heat transfer through slab.

Thermal Circuit




$$\Rightarrow q = \frac{T_1 - T_2}{\left(\frac{L}{kA}\right)} = \frac{T_2 - T_\infty}{\left(1/hA\right)}$$

$$\Rightarrow T_2 - T_\infty = \frac{q \times 1}{hA} = \frac{q_0}{h}$$

$$\Rightarrow T_2 = T_\infty + \frac{q_0}{h}$$

$$\frac{q}{A} \times \frac{L}{k} = T_1 - T_2$$

$$\Rightarrow \quad T_1 = \left(q_0 \times \frac{L}{k}\right) + T_2$$

$$\Rightarrow \quad T_1 = q_0\left(\frac{L}{k} + \frac{1}{h}\right) + T_\infty$$

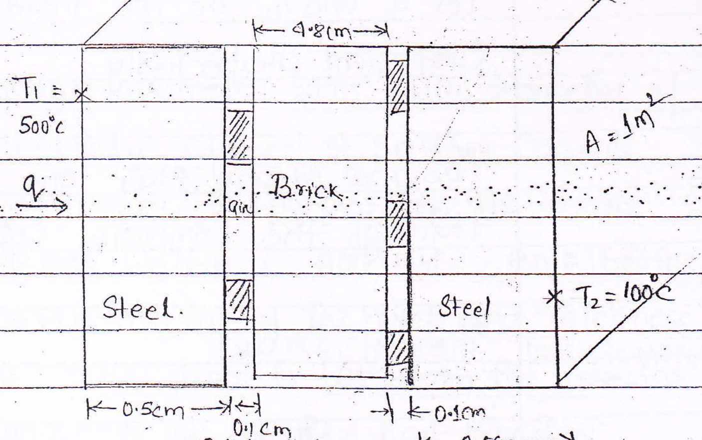
$$\Rightarrow \quad T_1 = 350^\circ C$$

$$T_2 = 250^\circ C$$

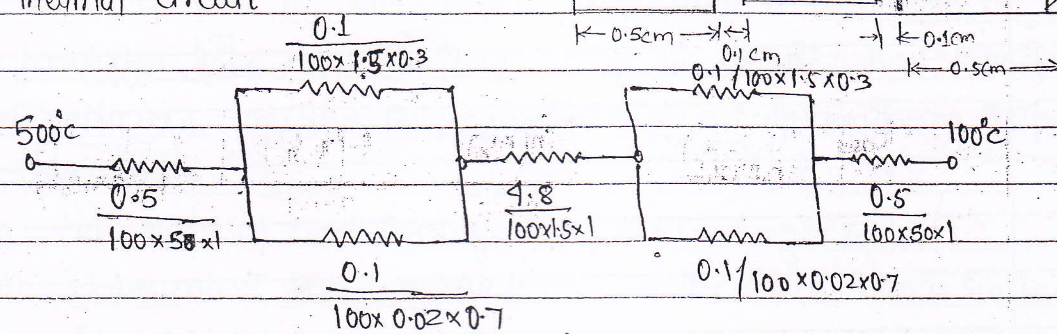
Ans.

IES-2001 A layer of 5 cm thick insulating brick having conductivity of 1.5 W/mK is placed b/w 2 0.5 cm thick steel plates, the conductivity of mild steel is 50 W/mK. The faces of brick adjacent to the plates are rough having solid to solid contact of 30% of the total area. The average height of the asperities is 0.1 cm. If the outer plate surface temperatures are 100°C and 500°C. Calculate the rate of heat transfer per unit area. The conductivity of air 0.02 W/mK. (25 Marks)

Ans. Assume steady state, one dimensional conduction heat transfer




Thermal Circuit

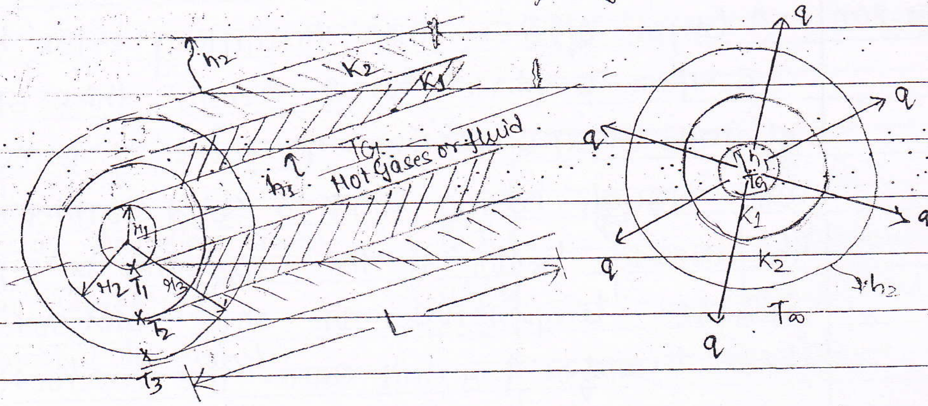


$$500^\circ C \;\; \frac{0.5}{100\times50\times1} \;\; 2.155\times10^{-3} \;\; \frac{4.8\times}{100\times1.5\times1} \;\; 2.155\times10^{-3} \;\; \frac{0.5}{100\times50\times1} \;\; 100^\circ C$$

$\Rightarrow \Sigma R_{th} = 0.0365$ K/Watt.

$$\Rightarrow q = \frac{500-100}{\Sigma R_{th}} = \frac{400}{0.0365} = 10.95\ kW/m^2$$

## Conduction - Convection Heat Transfer through a Composite System




Let $h_1$ and $h_2$ be the inside and outside Convective heat transfer Coefficient respectively.

Assume steady stat one dimensional radial heat transfer through the Composite Cylinder.

Thermal Circuit

$$T_{gases} \xrightarrow{q} \left(\frac{1}{h_1 2\pi r_1 L}\right) \; T_1 \; \frac{\ln(r_2/r_1)}{2\pi K_1 L} \; T_2 \; \frac{\ln(r_3/r_2)}{2\pi K_2 L} \; T_3 \; \left(\frac{1}{h_2 2\pi r_3 L}\right) \; T_\infty$$

$\Rightarrow$ Rate of heat transfer $q = \frac{T_{G} - T_{\infty}}{\Sigma R_{th}}$

$$= \frac{T_{G} - T_{\infty}}{\left(\frac{1}{h_1 2\pi r_1 L} + \frac{\ln\left(\frac{r_2}{r_1}\right)}{2\pi k_1 L} + \frac{\ln\left(\frac{r_3}{r_2}\right)}{2\pi k_2 L} + \frac{1}{h_2 2\pi r_3 L}\right)} \quad \text{--- (1)}$$

Since area of heat transfer is varying so we define two overall heat transfer Coefficients for inner and outer cylinder.

$\Rightarrow \theta = UA\Delta T$

$$= U_i A_i \Delta T = U_0 A_0 \Delta T$$

$$= U_i 2\pi r_1 L \Delta T = U_0 2\pi r_3 L \Delta T \quad \text{--- (2)}$$

$$\Rightarrow \frac{1}{U_i} = \frac{1}{h_1} + \frac{r_1}{k_1}\ln\frac{r_2}{r_1} + \frac{r_1}{k_2}\ln\frac{r_3}{r_2} + \frac{r_1}{r_3}\left(\frac{1}{h_2}\right)$$

$$\frac{1}{U_0} = \frac{r_3}{r_1}\left(\frac{1}{h_1}\right) + \frac{r_3}{k_1}\ln\left(\frac{r_2}{r_1}\right) + \frac{r_3}{k_2}\ln\frac{r_3}{r_2} + \frac{1}{h_2}$$

IES-2011 A steel pipe having internal diameter 2cm, outer diameter of 2.4cms and thermal conductivity of steel of 54 W/mK carries hot water at 95°c. Heat transfer Coefficient b/w the inner surface of steel pipe and hot water is 600 W/m²K. An asbestos insulation with thermal conductivity of 0.2 W/mK and thickness 2cm is put on the steel pipe. Heat is lost from the outer surface of asbestos insulated pipe ~~from~~ to the surrounding air at 30°C, heat transfer coefficient for the outer surface of insulation being 8 W/m²K.

Determine

(i) The rate of heat transfer per metre length of pipe.

(ii). Determine the temperatures at the inner, outer surfaces of the steel pipe and the outer surface of the insulation.

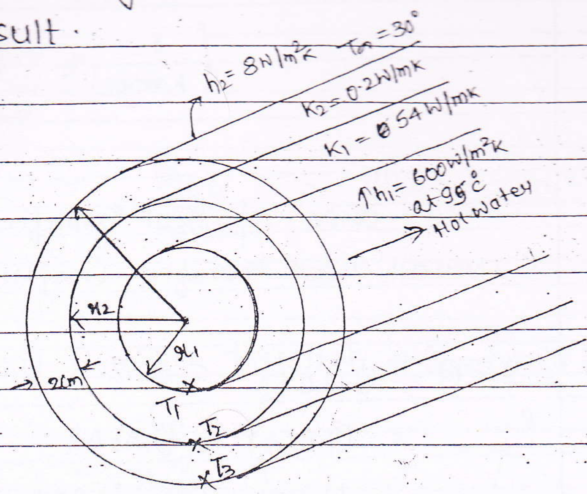
(iii). What do you understand by the term critical radius of insulation.

What is the value of Critical radius in above question. What is the ~~radi~~ rate of heat loss if thickness of insulation is Corresponding to Critical radius, Comments on the result.

Ans. $r_1 = 1\,cm$

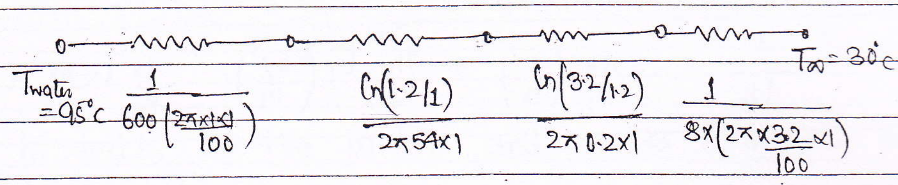
$r_2 = 1.2\,cm$

$r_3 = 3.2\,cm$




Assuming steady flow one dimensional radial Heat transfer.

Thermal Circuit




$$\Rightarrow q = \frac{95-30}{\Sigma R_{th}} = 45.4\ W/m$$

$$q = \frac{95-T_1}{\frac{1}{\left(600 \times 2\pi \times \frac{1}{100} \times 1\right)}} \Rightarrow T_1 = 93.78°C$$

$$q = \frac{93.78 - T_2}{\frac{\ln\left(\frac{1.2}{1}\right)}{2\pi 54 \times 1}} = 45.4 \Rightarrow T_2 = 93.76°C$$

$$\Rightarrow q = \frac{93.76 - T_3}{\frac{\ln\left(\frac{3.2}{1.2}\right)}{2\pi \times 0.2 \times 1}} = 45.4 \Rightarrow T_3 = 58.11°C$$

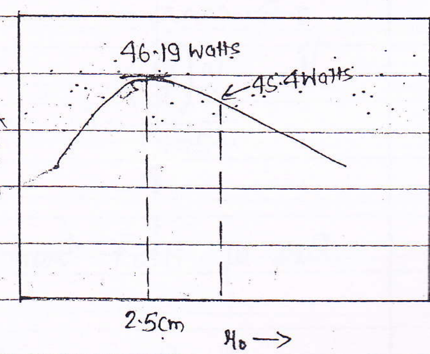
Very Small temp. drop across the steel pipe is due to very small thickness of pipe ($R_1 \rightarrow R_2$) and also due to high Conductivity of Steel. Very Small temp. drop b/w water and steel (inner surface) pipe is due to high Convective heat transfer Coefficient associated with water. High temperature drop across the insulation is due to its low Conductivity.

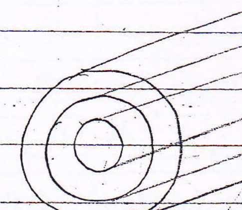
$$r_{crit} = \frac{K}{h} = \frac{0.2}{8} =$$

$$= 2.5\,cm$$

When $r_3 = 2.5\,cm$

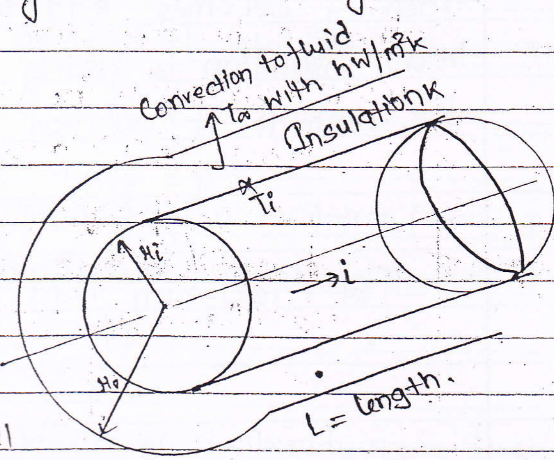
$q = 46.19$ Watts



The heat transfer rate from the hot water to the ambient can be reduced to below ~~45~~ 45.4 watts by adding so more ~~abe~~ asbestos insulation around the pipe also by changing the material to glass wool.

*** Critical Radius of Insulation.

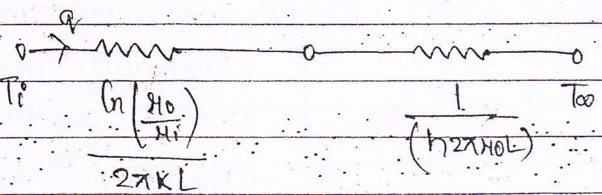



Consider a solid wire of radius $r_i$, inside which heat is being generated by passing electric current. Let an insulation having thermal

Conductivity K is wrapped around the wire ~~is radially~~ upto the radius $r_o$. The heat generated in the wire is radially Conducted through the insulation and then from the surface of the insulation. it is Convected to the ambient fluid at $T_\infty$ with a Convective heat transfer Coefficient of $h\ W/m^2K$.

Under steady state Conditions,
Let $T_i$ be the surface Temperature of wire

Drawing thermal Circuit b/w $T_i$ & $T_\infty$




Rate of Heat transfer b/w wire and fluid

$$= q = \frac{T_i - T_\infty}{\left(\frac{\ln \frac{r_o}{r_i}}{2\pi KL} + \frac{1}{h\, 2\pi r_o L}\right)} \text{ Watt.}$$

Keeping all other parameters like length of wire, Conductivity k, Convective Coefficient 'h' and radius of wire $r_i$ as Constant then q becomes a $f^n$ of $r_o$. The value of $r_o$ depends upon how much insulation is being wrapped around the wire. Greater the amount of insulation wrapped higher the '$r_o$' value.

$$q = f(r_o)$$

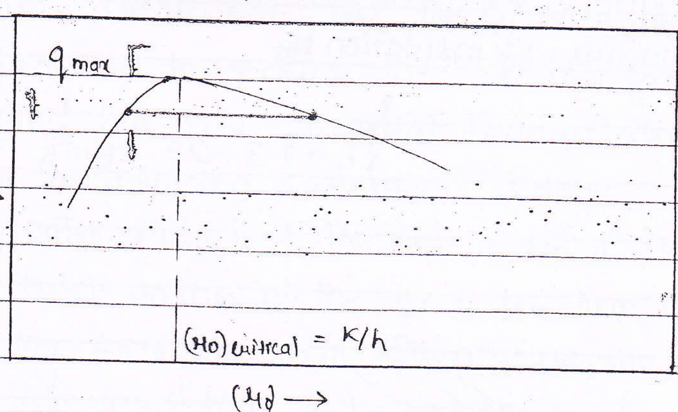
For Maximum Heat Transfer rate,

$$\frac{dq}{dr_o} = 0$$

$$\Rightarrow \frac{d}{dr_0}\left[\frac{T_i - T_\infty}{\left(\frac{\ln \frac{r_0}{r_i}}{2\pi K L} + \frac{1}{h 2\pi r_0 L}\right)}\right] = 0$$

$$\Rightarrow r_0 = \frac{K}{h} \quad \text{(Critical radius of Insulation)}$$

## Physical Significance of Critical radius of Insulation




For sufficiently thin wires whose radius is lesser than critical radius of insulation any insulation wrapped around it shall increase the heat transfer rate instead of decreasing it, this happens so, because when more and more insulation is being wrapped ~~over~~ around wire there is a rapid ~~decrease~~ increase of convection resistance as compared to the slight ~~increase~~ increase of conductance resistance. The overall combined effect being decrease of total resistance and increase of heat transfer rate.

This continues to happen upto critical radius of insulation (where $q$ is max) beyond which any further insulation added shall decrease the heat transfer rate.

In case if the radius of the wire chosen is already more than the

Critical radius of insulation, any insulation put up around it shall decrease the heat transfer rate.

from the fig. we can analyse that when the radius of wire is below

1/11/2011 Critical radius then there exists two value of q with and without insulation.

GATE-2000 A steel steam pipe 10cm inner diameter and 11cm outer diameter is covered with an insulation having a thermal conductivity of 1 W/mK. If the convective heat transfer coeff. b/w the surface of ~~air~~ insulation and surr. air is 8 W/m²K. The critical radius of insulation is

(a) 10cm

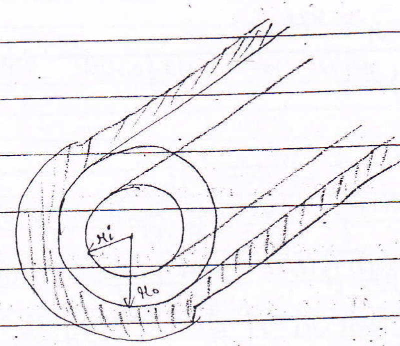
(b) 11cm

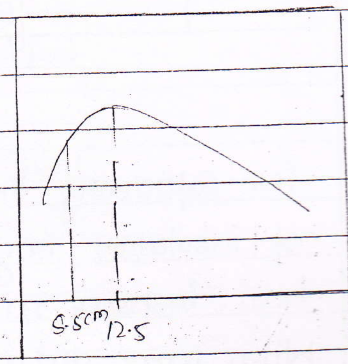
(c) 15cm

(d) 12.5cm

{$h \rightarrow 3-23$ W/m²K free convection}

Ans.




$$r_{crit} = \frac{K_{Ins}}{h} = 12.5cm$$

To prevent condensation do not keep insulation.

GATE-1999 It is proposed to coat a 1mm diameter wire with enamel paint (K = 0.1 W/mK) to increase heat transfer with air if the air side heat transfer coefficient is 100 W/m²K, the optimum thickness of enamel paint should be

(a) 0.25 mm (b) 0.5 mm (c) 1mm (d) zero.

Ans. $r_{crit} = \frac{0.1}{100} = 1\,mm$

$r_i = 0.5\,mm$

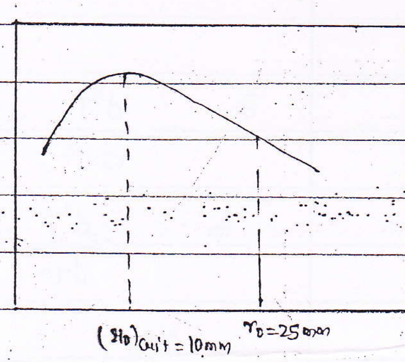
$\therefore$ Thickness of insulation $= 1 - 0.5 = \underline{\underline{0.5\,mm}}$ Ans.

GATE-2011: A pipe of 25mm outer diameter carries steam, the heat transfer coefficient b/w the cylinder and the surr. is $5\,W/m^2K$. It is proposed to reduce the heat loss from the pipe by adding insulation having a thermal conductivity of $0.05\,W/mK$. Which one of the following statements is true.

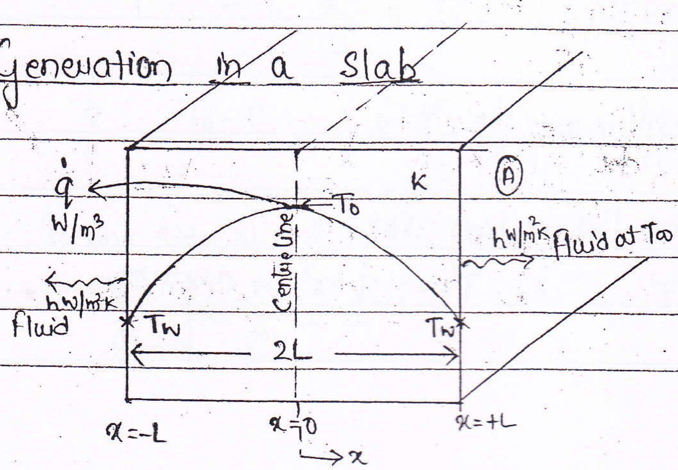
(a). The outer radius of the pipe is equal to critical radius.
(b). The outer radius of the pipe is less than critical radius.
(c). Adding insulation will ~~not~~ increase the heat loss.
(d). Adding insulation will reduce the heat loss.

Ans. $r_{crit} = \frac{0.05}{5} = 10\,mm$

We are on right side of the graph, So, ~~Crit~~ insulation will reduce the heat loss.




## Heat Generation in a slab



Consider a slab of thickness '2L' inside which there is uniform heat generation of $\dot{q}$ (W/m³) throughout the slab

Assumptions

o Steady state heat transfer $T \neq f(t)$
To satisfy this condition heat must be convected either from both sides of the slab to a fluid or atleast from one side of the slab.

o One dimensional conduction
$T = f(x)$ only

o Uniform $\dot{q}$ i.e. $\dot{q} \neq f(x)$

o $K = C$ i.e. material is isotropic

General 3-D heat conduction eqn:

$$\frac{\partial^2 T}{\partial x^2} + \cancelto{0}{\frac{\partial^2 T}{\partial y^2}} + \cancelto{0}{\frac{\partial^2 T}{\partial z^2}} + \frac{\dot{q}}{K} = \cancelto{0}{\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

$$\Rightarrow \frac{d^2 T}{dx^2} + \frac{\dot{q}}{K} = 0$$

$$\Rightarrow \frac{d^2 T}{dx^2} = -\frac{\dot{q}}{K}$$ (Poisson's eqn in one dimension)

Integrating w.r.t x

$$\Rightarrow \frac{dT}{dx} = -\frac{\dot{q}x}{K} + C_1$$

Again Integrating w.r.t x

$$\Rightarrow T = -\frac{\dot{q}}{K}\frac{x^2}{2} + C_1 x + C_2$$

$C_1$ and $C_2$ are Constants of integration that are to be obtained from boundary Conditions.

One special boundary Cond$^n$ is

At $x = L$, $T = T_w$

At $x = -L$, $T = T_w$

This can be possible only when both sides of the slab are subjected to the same fluid at the same temperature with the same Convective heat transfer Coefficient.

To satisfy this boundary condition the value of $C_1$ must be zero

$$\Rightarrow \quad T = -\frac{\dot{q}}{K}\frac{x^2}{2} + C_2$$

Temperature will be maximum only when

$$\frac{dT}{dx} = 0$$

$$\Rightarrow \quad 0 = -\frac{\dot{q}}{K}x \quad \Rightarrow \quad x = 0$$

$\Rightarrow$ T is maximum at centre line.

Let the maximum temp. be $T_0$ i.e. at $x = 0$

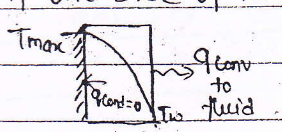
$$\Rightarrow \quad T_0 = C_2$$

The temperature distribution is

$$T = -\frac{\dot{q}}{K}\frac{x^2}{2} + T_0$$

$$\Rightarrow \quad T_0 - T = \frac{\dot{q}}{K} \times \frac{x^2}{2} \quad \text{(Parabolic)}$$

Note :- If one side of the slab is insulated $\Rightarrow q_{cond} = 0 \Rightarrow -KA\frac{dT}{dx} = 0$




$\Rightarrow \frac{dT}{dx} = 0 \Rightarrow$ T is max.

Put $x = +L$ or $x = -L \Rightarrow T = T_w$

$$\Rightarrow \quad T_0 - T_w = \frac{\dot{q}L^2}{2k}$$

$$\Rightarrow \quad \underset{\substack{\text{at}\\\text{Centre}\\\text{line}}}{(T_0)} = T_w + \frac{\dot{q}L^2}{2k}$$

To get $T_w$ :-

For steady state conditions, Energy balance :-

Total heat generated in the slab = Total heat convected from both sides of slab

$$\Rightarrow \quad \dot{q} \times (\cancel{2}L \times \cancel{A}) = \cancel{2} \times h \times \cancel{A}\,(T_w - T_\infty)$$

The above two are equated as we want to get Temp relation with respect to ambient Temp.

$$\Rightarrow \quad \frac{\dot{q} \times L}{h} = T_w - T_\infty$$

$$\Rightarrow \quad T_w = T_\infty + \frac{\dot{q}L}{h}$$

$$\Rightarrow \quad T_0 = T_\infty + \frac{\dot{q}L}{h} + \frac{\dot{q}L^2}{2k}.$$

GATE- 2007 Consider steady one dimensional heat flow in a plate of 20 mm thickness with a uniform heat generation of 80 MW/m³ the left and right faces are kept at constant temperatures of 160°c and 120°c resp. The plate has a constant thermal conductivity of 200 W/mK. The location of maximum temp. within the plate from its left face is

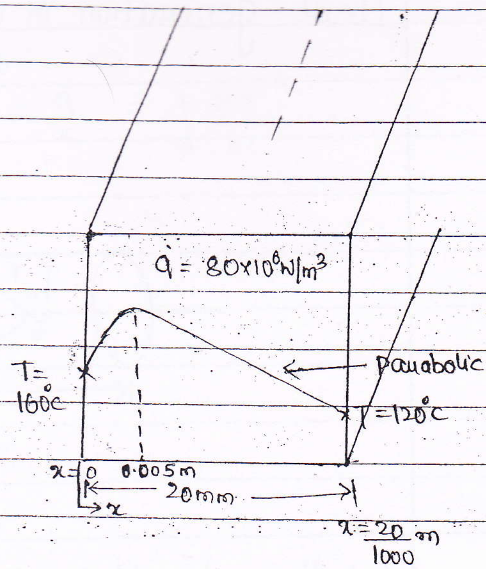
(a) 15 mm (b) 10 mm (c) 5 mm (d) zero.

The maximum temp. within the plate in degree celcius is

(a) 160 (b) 165 (c) 200 (d) 250

Ans. General eq$^n$ is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$




Integrating

$$\frac{dT}{dx} = -\frac{\dot{q}x}{k} + C_1$$

Again, Integrating

$$T = -\frac{\dot{q}}{k}\frac{x^2}{2} + C_1x + C_2$$

At $x = 0 \Rightarrow T = 160°c$

$$160 = -\frac{\dot{q}}{k}\frac{0^2}{2} + C_1(0) + C_2$$

$$\Rightarrow C_2 = 160$$

At $x = 0.02$ m

$$\Rightarrow 120 = -\frac{\dot{q}}{k}\frac{(0.02)^2}{2} + \overset{C_1}{\cancel{160}}\times(0.02) + \cancel{C_2}\ 160$$

$$\Rightarrow C_1 = 2000$$

$$\Rightarrow T = -\frac{\dot{q}}{k}\frac{x^2}{2} + 2000x + 160 \quad \text{——— (1)}$$

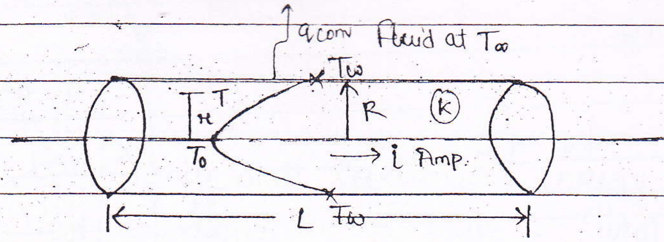
T is maximum when $\frac{dT}{dx} = 0$

$$\Rightarrow -\frac{\dot{q}x}{k} + 2000 = 0$$

$$\Rightarrow x = \frac{2000\times200}{80\times10^6} \quad \Rightarrow x = 5\text{mm Ans.}$$

$$\therefore T_{max} = -\frac{\dot{q}}{k}\frac{(0.005)^2}{2} + 2000\times(0.005) + 160 = 165°c$$

# Heat generation in a Solid Cylinder




$$T = f(r)$$

Let there be Uniform heat generation in the wire by passing electric Current.

Assume steady state Conditions

To satisfy this,

Heat generated in the wire = Heat Convected to fluid

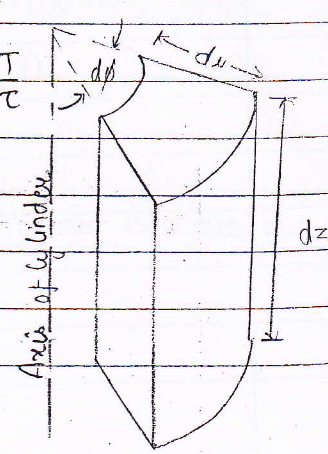
Assume One dimensional Radial Conduction $T = f(r)$

$\dot{q} = C \quad K = C$

To get $T = f(r)$

General Conduction eq$^n$ in Cylindrical Co-ordinates

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{K} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$




$\phi \rightarrow$ Azimuthal (Angular)

$r \rightarrow$ Radial

$z \rightarrow$ axial

One dimensional unsteady state heat transfer eqⁿ for a sphere with heat generation at the rate of 'q'.

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)+\frac{q}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

by assumptions,

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\cancel{\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}}^{0}+\cancel{\frac{\partial^2 T}{\partial z^2}}^{0}+\frac{\dot{q}}{k}=\cancel{\frac{1}{\alpha}\frac{\partial T}{\partial \tau}}^{0}$$

$$\Rightarrow \frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\dot{q}}{k}=0$$

$$\Rightarrow r\frac{\partial^2 T}{\partial r^2}+\frac{\partial T}{\partial r}=-\frac{\dot{q}}{k}\times r$$

$$\Rightarrow \frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{\dot{q}}{k}r$$

On Integration,

$$r\frac{dT}{dr}=-\frac{\dot{q}}{k}\times\frac{r^2}{2}+C_1$$

$$\Rightarrow \frac{dT}{dr}=-\frac{\dot{q}}{k}\frac{r}{2}+\frac{C_1}{r}$$

Again Integrating,

$$T=-\frac{\dot{q}r^2}{4k}+C_1\ln r+C_2$$

One boundary Condⁿ is at $r=R$, $T=T_w$ (surface temp of wire)

The other boundary condⁿ is for steady state condⁿs,

heat generated in the wire = Heat conducted at the surface

$$\Rightarrow \dot{q}\times\cancel{\pi}R^{\cancel{2}}\cancel{L}=-(k\,2\cancel{\pi}\cancel{R}\cancel{L})\left(\frac{dT}{dr}\right)_{r=R}$$

$$\Rightarrow \left(\frac{dT}{dr}\right)_{r=R}=-\frac{\dot{q}R}{2k} \quad \text{———} \quad (1)$$

$$\frac{dT}{dr} = -\frac{\dot{q}}{k}\frac{r}{2} + \frac{C_1}{r}$$

$$\text{at } \left(\frac{dT}{dr}\right)_{r=R} = -\frac{\dot{q}}{k}\frac{R}{2} + \frac{C_1}{R} \quad \text{———— (II)}$$

from (I) and (II)

$$\Rightarrow C_1 = 0$$

$C_1$ can also be told as zero because logarithmic fn becomes infinte at $r = 0$ i.e. axis of Wire

T is maximum when $\frac{dT}{dr} = 0$

$$\Rightarrow \quad 0 = -\frac{\dot{q}r}{2k} + \frac{C_1}{r}$$

$$\Rightarrow \quad r = 0$$

hence, $T_{max}$ at the axis of wire.

Let the maximum temp. be $T_0$

$$T = -\frac{\dot{q}r^2}{4k} + C_1 \cancel{\ln r}^{\,0} + C_2$$

Put $r = 0$

$$\Rightarrow \quad C_2 = T_0$$

$$\Rightarrow \quad T = -\frac{\dot{q}r^2}{4k} + T_0$$

$$\Rightarrow \quad T_0 - T = \frac{\dot{q}r^2}{4k}$$

Put $r = R \Rightarrow T = T_w$

$$\Rightarrow T_0 - T_w = \frac{\dot{q}R^2}{4k}$$

$$\Rightarrow \underset{\text{i.e. at axis}}{(T_0)} = T_w + \frac{\dot{q}R^2}{4k}$$

To get $T_w$

Writting the Energy balance, equation for steady state

$\Rightarrow$ Heat generated = Heat Convected

$$\Rightarrow \dot{q} \times \pi R^2 L = h\, 2\pi R L \,(T_w - T_\infty)$$

$$\Rightarrow T_w = \frac{\dot{q}R}{2h} + T_\infty$$

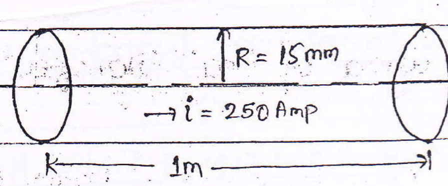
$$\Rightarrow T_0 - T_w = \frac{\dot{q}R^2}{4K}$$

$$\Rightarrow (T_0)_{\text{i.e. at axis}} = T_w + \frac{\dot{q}R^2}{4K}$$

$$\Rightarrow (T_0)_{axis} = \frac{\dot{q}R}{2h} + T_\infty + \frac{\dot{q}R^2}{4K}$$

Q. A Copper Cable of 30 mm diameter has an electrical resistance of $5 \times 10^{-3}$ Ω/m and is used to Carry an electrical Current of 250 Amp. The cable is exposed to ambient air at 20°c and the associated Convection Co-efficient is 25 W/m²K. What are the surface and Centre line temperatures of wire.

Ans Let us Consider a unit length wire.



Heat generated per unit volume $= \dot{q} = \dfrac{i^2 R_{ele}}{\pi R^2 L}$ W/m$^3$

$$= \frac{250^2 \times 5\times10^{-3}}{\pi \times (15/1000)^2 \times 1} \text{ W/m}^3 = \cancel{17698 \text{ W/m}^3}\ 442097 \text{ W/m}^3$$

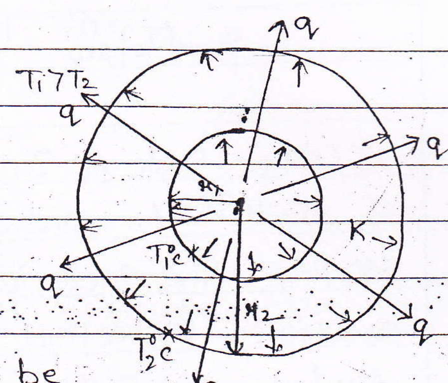
$K_{cu} = 390$ W/mK

$\Rightarrow (T_0)_{axis} = 152.99°c$

$T_w = 152.68°c$

omment :- The very small temp. difference b/w centre line and the surface of the wire is due to very high conductivity of copper. If water is kept around the cable at the same temp. of 20°c, the surface temp may fall down to less than 25°c.

## Conduction heat transfer through hollow sphere




The dirn of h.t is from in to out.

Consider a hollow sphere of inner radius $r_1$ and outer radius $r_2$.
Let the inside spherical surface be maintained at $T_1°c$ and outside surface temp. $T_2°c$. Conduction heat transfer occurs radially outwards from inside surface at $T_1$ to outside surface at $T_2$.
Here, the conduction area of heat transfer changes in the dirn of heat flow.
Assume steady state one dimensional radial H.T with constant 'K' and no heat generation.
At any radius $r$,

Area of Conduction H.T = $4\pi r^2$

By Fourier' law of Conduction

$$q = \text{Rate of Heat transfer} = \left(-KA\frac{dT}{dx}\right)$$

$$\Rightarrow q = -K\,4\pi r^2 \frac{dT}{dr}$$

$$\Rightarrow q\frac{dr}{r^2} = -4\pi K\, dT$$

$q \neq f(r)$ [To maintain steady state]

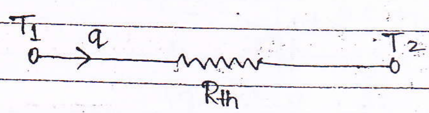
$$\Rightarrow \int_{r_1}^{r_2} q\frac{dr}{r^2} = \int_{T_1}^{T_2} -4\pi K\, dT$$

$$\Rightarrow q\left[-\frac{1}{r}\right]_{r_1}^{r_2} = 4\pi K(T_1 - T_2)$$

$$\Rightarrow q\left[\frac{1}{r_1} - \frac{1}{r_2}\right] = 4\pi K(T_1 - T_2)$$

$$\Rightarrow q = \frac{4\pi K(T_1 - T_2) \times r_1 \times r_2}{(r_2 - r_1)}$$

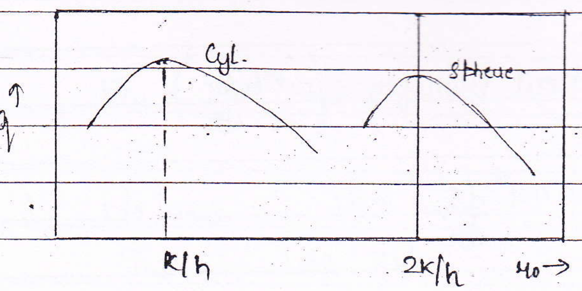
Equivalent Thermal Circuit




$$\Rightarrow (R_{th})_{sph} = \frac{T_1 - T_2}{q} = \frac{r_2 - r_1}{4\pi K r_1 r_2}$$

$$(r_{crit})_{sphere} = \frac{2K_{insulation}}{h}$$

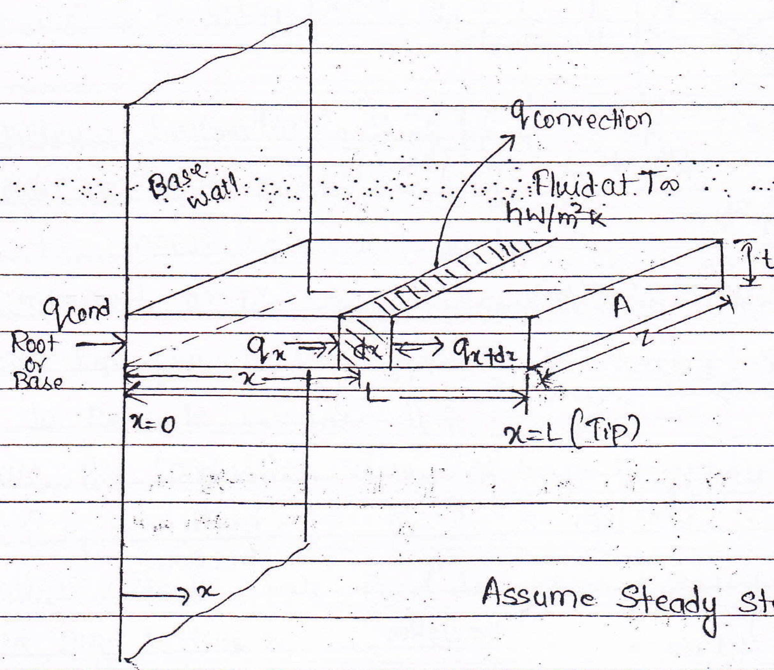
Spherical vessels are used for storage of Cryogenic liquid as for a sphere, for given volume it has the least area, so less heat transfer.




## Fins (Extended Surfaces)

Fins are the projections protruding from a hot surface and they are meant for increasing heat transfer rate by increasing surface area of heat transfer.

### Analysis of Rectangular fin




Assume Steady state Conditions.

Heat is Conducted from the base wall at $T_0$ into the fin at its root and then heat is simultaneously Conducted along the length of fin in the x-dirxⁿ and heat is also Convecting from the surface of the fin into the fluid at $T_\infty$ with Convection Coefficient of h W/m²K.

Consider a small element of fin of length 'dx' at a distance of x from the root.

Let $q_x$ = Heat Conducted into the element along x-dirxⁿ

$$= -KA\frac{dT}{dx}$$

$\Rightarrow$ $q_{x+dx}$ = Heat Conducted out of the element

$$= q_x + \frac{\partial}{\partial x}(q_x)\,dx$$

$\Rightarrow$ $q_{\text{convected from the element}} = hP\,dx\,(T - T_\infty)$

Where P is perimeter of the fin $= 2(z+t)$

Writing the energy balance for steady state

$$\Rightarrow q_x = q_{x+dx} + q_{conv}$$

$$\Rightarrow \cancel{q_x} = \cancel{q_x} + \frac{\partial}{\partial x}(q_x)\,dx + q_{conv}$$

$$\Rightarrow 0 = \frac{\partial}{\partial x}(q_x)\,dx + hp\,dx\,(T - T_\infty)$$

$$\Rightarrow 0 = \frac{\partial}{\partial x}\left(-KA\frac{dT}{dx}\right)\cancel{dx} + hp\,\cancel{dx}\,(T - T_\infty)$$

$$\Rightarrow \frac{d^2T}{dx^2} - \frac{hP}{KA}(T - T_\infty) = 0$$

Put $T - T_\infty = \theta$

$$\Rightarrow \frac{dT}{dx} = \frac{d\theta}{dx} \Rightarrow \frac{d^2T}{dx^2} = \frac{d^2\theta}{dx^2}$$

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Put $m^2 = \frac{hP}{KA}$

$$\Rightarrow \frac{d^2\theta}{dx^2} - m^2\theta = 0$$

This a standard format of D.E, whose soln is given by,

$$\theta = C_1 e^{-mx} + C_2 e^{mx}$$

$$m = \sqrt{\frac{hP}{KA}} \;/\text{metre}$$

$C_1$ & $C_2$ are constants of integration that are to be obtained from boundary conditions.

One common boundary condn is
At $x = 0 \Rightarrow T = T_0$
$\Rightarrow \theta = T_0 - T_\infty = \theta_0$

The other boundary condn depends on 3 different cases.
Case(i)
Fin is Infinitely Long (very long fin)

Then the temp. at the tip of the fin will be essentially that of the fluid ($T_\infty$)
i.e At $x = \infty \Rightarrow T = T_\infty \Rightarrow \theta = 0$

Then the soln is
$$\frac{\theta}{\theta_0} = \frac{T - T_\infty}{T_0 - T_\infty} = e^{-mx}$$

Then the heat transfer rate through the fin is
$$q_{fin} = -KA\left(\frac{dT}{dx}\right)_{x=0} = \text{Heat Conducted into the fin.}$$

$$\Rightarrow \quad q_{fin} = \sqrt{hPKA}\ \theta_0$$

$$= \sqrt{hPKA}\ (T_0 - T_\infty) \text{ Watt.}$$

Case (ii)

Fin Tip is Insulated

i.e. due to small x-sectional area and also considerable temp. drop $T_w - T_\infty$ is very small so considered as insulated.

$$q_{cond.} \text{ at } x = L = 0$$

$$\Rightarrow \quad -KA\left(\frac{dT}{dx}\right) = 0$$

$$\Rightarrow \left(\frac{dT}{dx}\right)_{x=L} = 0 \quad \Rightarrow \left(\frac{d\theta}{dx}\right)_{x=L} = 0$$

Then the soln is

$$\frac{T - T_\infty}{T_0 - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}$$

Then the rate of heat transfer through fin is

$$q_{fin} = -KA\left(\frac{dT}{dx}\right)_{x=0}$$

$$= \sqrt{hPKA}\ \theta_0 \tanh mL \text{ Watts}$$

Case (iii)

Fin is finite in length & also loses heat by convection from its tip

The soln is

$$\frac{T-T_\infty}{T_0-T_\infty} = \frac{\theta}{\theta_0} = \frac{\cosh m(L_c-x)}{\cosh mL_c}$$

Where $L_c$ = Corrected length = $\left(L+\frac{t}{2}\right)$

$$q_{fin} = \sqrt{hPKA}\ \theta_0 \tanh mL_c$$

However, the Case (ii) and case (iii) has only 1% error.

(Case iv) If its a pin fin

$$\Rightarrow L_c = L + \frac{d}{4}$$

Note:- If no fin case is mentioned in any problem by default assume insulated tip case.

12|2011
(~~Case iv~~)
A-TE-2010 A fin has 5mm diameter and 100mm length. The thermal conductivity of fin material is 400 W/mK. One end of the fin is maintained at 130°c and its remaining surface is exposed to ambient air at 30°c. If the convective heat transfer coeff. is 40 W/m²K, the heat loss in Watts from the fin is

(a). 0.08  (b) 5.0  (c) 7.0  (d) 7.8

Ans. $m = \sqrt{\frac{hP}{KA}}$

$= \sqrt{\frac{40 \times \pi \times 5 \times 10^{-3} m}{400 \times \frac{\pi}{4} (5 \times 10^{-3})^2}} = 0.283 \times \sqrt{1000} = 8.94 /m$

$\Rightarrow q_{fin} = \sqrt{hPKA} \times (130-30) \times \tanh(\cancel{0.283 \times 100}\ 8.94 \times 0.1)$

$= \underline{\underline{5.01 W}}$ Ans..

I.E.S 2004 An electronic Semiconductor device generates heat equal to $480 \times 10^{-3}$ W in order to keep the Surface temperature at the upper Safe limit of 70°c the generated heat has to be dissipated to the Surrounding which is at 30°c. To accomplish this task aluminium fin of 0.7 mm square and 12 mm long is attached to the surface. The thermal Conductivity of aluminium fins is 170 W/mk. If the heat transfer Coeffient is 12 W/m²K, Calculate the no. of fins required, assume no heat loss from tip of the ~~Case~~. fin.

Ans. This is a Case (ii) problem of fin tip insulated.

$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{12 \times 2(0.7+0.7) \times 10^{-3}}{\cancel{48}\ 170 \times (0.7 \times 10^{-3})^2}}$

$= 20.08 /m$

$q_{fin} = \sqrt{hPKA} \times \theta_0 \times \tanh mL$

$= \sqrt{12 \times 2.8 \times 10^{-3} \times 170 \times (0.7 \times 10^{-3})^2} \times (70-30) \times \tanh(20.08 \times 0.012)$

$= 0.0158 W$

let there be $n$ fins reqd.

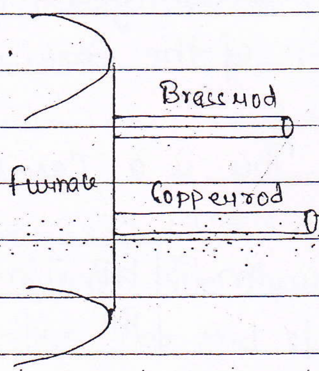
$$\Rightarrow \quad n \times 0.0158 = 480\times10^{-3}$$

$$\Rightarrow \quad n = 30.37 \simeq 30 \text{ fins.}$$

as some heat loss will be accomodated by bare area which will compensate for 0.37 fins.

IES 1997 Two long rods of the same diameter, one made of brass ($K = 85\,W/mK$) and the other made of Copper ($K = 375\,W/mK$) have one of their ends inserted into a furnace. Both rods are exposed to the same environment. At a section 10.5 cm away from the furnace end, the temperature of the brass rod is 120°C. At what distance from the furnace end, the same temp. would be reached in the copper rod.

Ans. long ⇒ Case(i) Infinitely long fin.




$$m_{brass} = \sqrt{\frac{h_b P_b}{K_b A_b}}$$

let '$D_b$' be the diameter of brass rod.

$$\Rightarrow \quad P_b = \pi D_b$$

$$A_b = \frac{\pi}{4} D_b^2$$

$$\Rightarrow \quad m_{brass} = \sqrt{\frac{h_b \times 4}{85 \times D_b}}$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-m_b L_b}$$

$$\Rightarrow 120 = T_\infty + (T_0 - T_\infty) e^{-m_b l_b} \quad \text{—— (I)}$$

for Copper,

$$120 = T_\infty + (T_0 - T_\infty)^{-m_c L_c} \quad \text{—— (II)}$$

$$m_{copper} = \sqrt{\frac{h_c \times 4}{375 \times D_c}}$$

$h_c = h_b$ (same environment) $= h$

$D_c = D_b$ (same diameter)

equating (I) and (II)

$$\Rightarrow \cancel{T_\infty} + (T_0 - \cancel{T_\infty}) e^{-m_b l_b} = \cancel{T_\infty} + (T_0 - \cancel{T_\infty}) e^{-m_c L_c}$$

$$\Rightarrow e^{-m_b l_b} = e^{-m_c l_c}$$

Taking natural logarithm on both sides

$$\Rightarrow m_b l_b = m_c l_c$$

$$\Rightarrow \sqrt{\frac{\cancel{h} \times \cancel{4}}{85 \times \cancel{D_b}}} \times 10.5\,cm = \sqrt{\frac{\cancel{h} \times \cancel{4}}{375 \times \cancel{D_c}}} \times L_c$$

$$\Rightarrow l_c = \sqrt{\frac{375}{85}} \times 10.5\,cm \Rightarrow L_c = \underline{\underline{22.05\ cm}}$$

Copper is offering lesser conduction resistance to the heat flow as compared to brass.

## Fin Efficiency ($\eta_{fin}$)

It is defined as the ratio b/w actual heat transfer rate taking place through the fin and the maximum possible heat transfer rate that could occur through the fin i.e. the entire fin surface is at its root temperature.

The entire fin surface will be at its root temperature $T_0$ only when the material of the fin has infinite thermal conductivity k.

$$\eta_{fin} = \frac{q_{act}}{q_{maximum\ possible}}$$

Since more important is case (ii)

$$\Rightarrow \eta_{fin} = \frac{\sqrt{hPKA}\ \theta_0 \tanh mL}{h(PL)(T_0 - T_\infty)}$$

$$= \frac{\tanh mL}{\sqrt{\frac{hP}{KA}} \times L} = \frac{\tanh mL}{mL}$$

$$\eta_{fin} \propto \sqrt{K} \quad (h, P, A \text{ Constant})$$

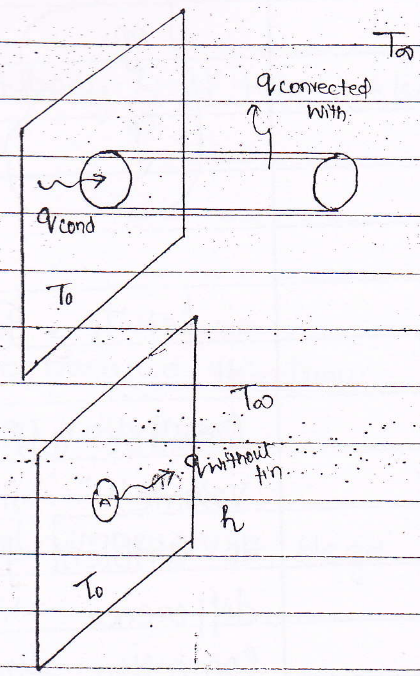
K must be high to have higher $\eta$

Ex:- Al, Cu.

## Fin Effectiveness ($\varepsilon_{fin}$)

It is defined as the ratio b/w heat transfer rate with fin and the heat transfer rate without fin. This parameter tells about whether the fins are really useful in increasing the heat transfer rate for a particular application. fins are desired only when $\epsilon_{fin}$ is more than 4 and 5.

$$\epsilon_{fin} = \frac{q_{with\,fin}}{q_{without\,fin}}$$




$$= \frac{\sqrt{hPKA}\,\theta_0 \tanh mL}{hA(\cancel{T_0 - T_\infty})}$$

$$\Rightarrow \epsilon_{fin} = \frac{\tanh mL}{\sqrt{\frac{hA}{KP}}}$$

$$\Rightarrow \epsilon_{fin} \propto \frac{1}{\sqrt{h}}$$

i.e. If $h$ is higher effectiveness of fin is low.

fins are really helpful in increasing the heat transfer rate particularly when $h$ values are low i.e. free convection in air (then effectiveness are higher), ∴ fins are not generally used with water.

$$\epsilon_{fin} \propto \sqrt{\frac{P}{A}}$$

$A \leftarrow$ Profile area

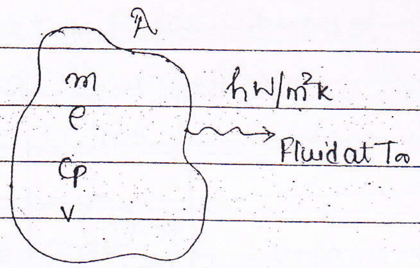
Generally fins must be of large $\left(\frac{P}{A}\right)$ ratio i.e. thin fins having a large perimeter many in number are generally used.

also $\epsilon_{fin} \propto \sqrt{K}$

∴ high $K$ leads to both high $\eta_{fin}$ & $\epsilon_{fin}$

# Unsteady State OR Transient Conduction Heat Transfer

Temperature of body Changes With Time.




Let $T_i$ = Initial temp. of body when time = 0 sec.

Consider a body of mass 'm', density '$\rho$', Specific heat '$C_p$', and Volume 'V' which is at initial temp. of $T_i$ is suddenly exposed to ambient fluid at $T_\infty$ (a thermal reservoir), due to temp difference with fluid the body keeps on loosing heat by Convection ~~due~~ to the fluid with a Convective heat transfer Coefficient of $h\ W/m^2K$. As a result, the internal energy of body keeps on decreasing with time which is manifested (shown) by decrease of temp.

Let T = Temperature of body at any instant of time $\tau$ secs measured from the instant $\tau = 0$ sec. (When body is suddenly exposed)

$T = f(\text{time})$

As $\tau$ increases $\Rightarrow$ T decreases.

Writing the energy balance eq$^n$ for the body at any instant of time $\tau$ sec, We write

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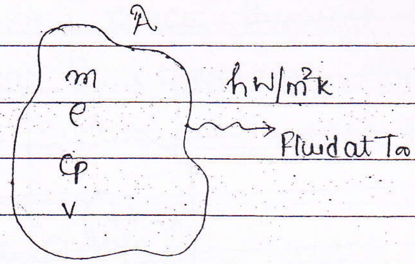
# Unsteady State OR Transient Conduction Heat Transfer

Temperature of body Changes With Time.

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Let T = Temperature of body at any instant of time τ secs measured from the instant τ = 0 sec. (When body is suddenly exposed)

$T = f(\text{time})$

As τ increases ⇒ T decreases.

Writing the energy balance eqⁿ for the body at any instant of time τ sec, we write the rate of Convection heat loss from body to fluid = Rate of decrease in Internal energy of body.

$$\Rightarrow \quad hA(T-T_\infty) = -mC_p\left(\frac{dT}{d\tau}\right)$$

$\frac{dT}{d\tau}$ = Rate of Cooling or heating of body w.r.t time in k/sec.

$$\Rightarrow \quad hA(T-T_\infty) = -\rho V C_p\left(\frac{dT}{d\tau}\right)$$

The rate of Cooling also changes with time because the temp. difference b/w body and fluid keeps changing with time.

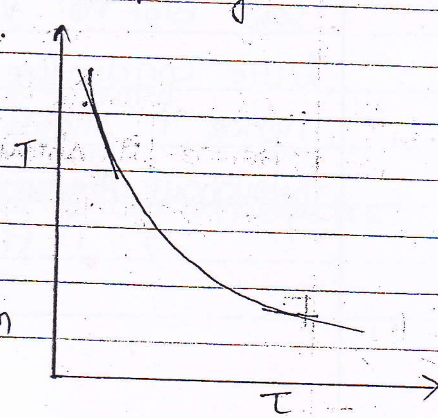
Separating the variables, Time and temperature, we get,

$$\int_0^{\tau\,sec} \frac{hA}{\rho V C_p}\, d\tau = \int_{T_i}^{T} \frac{-dT}{(T-T_\infty)}$$

$$\Rightarrow \left(\frac{hA}{\rho V C_p}\right)\tau = \ln\left(\frac{T_i-T_\infty}{T-T_\infty}\right)$$

$$\Rightarrow \boxed{\frac{T_i-T_\infty}{T-T_\infty} = e^{\left(\frac{hA}{\rho V C_p}\right)\tau}}$$

$\left(\frac{\rho V C_p}{hA}\right)$ is Called Time Constant of body since it has Units of sec.




Initially the rate of Cooling of the body is very high since the temp. difference b/w body and fluid is also very high but as time progresses it decreases as shown in the graph.

When area decreases Lumped heat capacity increases i.e. Temp in

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**Note:-** In the above analysis made it is assumed that the internal temp. gradients within the body are neglected i.e. the body's temp. is Uniform through out. Such analysis is called lumped heat capacity analysis for which the conductivity of the body must be high also the size of the body is relatively small.

## Criteria for lumped heat capacity analysis

Biot No. < 0.1

$$\text{Biot No} = \frac{hs}{K_{solid}}$$

Where S = Characteristic dimension of body.

$$= \frac{V}{A}$$

For sphere,

$$S = \frac{\frac{4}{3}\pi R^3}{4\pi R^2} = \frac{R}{3}$$

$$\text{Biot No.} = \frac{(S/KA)}{(1/hA)} = \frac{\text{Internal Conductive resistance offered by body}}{\text{Surface Convective Resistance}}$$

$$= \frac{ICR}{ECR}$$

Low Biot no. values indicate that the body is offering very little conductive resistance due to its high conductivity hence it shows uniform temperature or same temp. throughout the body at any instant of time.

$\Rightarrow T = f(\tau)$ &

$T \neq f(\text{space})$

Biot No. resembles Nusselt No.

$$Nu = \left(\frac{hD}{K_{fluid}}\right) \text{ in Convective heat transfer}$$

Nusselt no. is always > 1.

G.A.T.E-2011 A spherical steel ball of 12mm diameter is initially at the 1000K, it is slowly cooled in a surrounding of 300K, the heat transfer coefficient b/w the steel ball and the surr. is $5 W/m^2K$. The thermal conductivity of steel is 20W/mK. The temp. difference b/w the centre and the surface of steel ball is large because

(a) Conduction resistance is far higher than convective resistance.

(b) Large because conduction resistance is far lesser than convective resistance.

(c). Small because Conduction resistance is far higher than the convective resistance

(d). Small because Conduction resistance is far less than the convective resistance.

Ans. 'S' for sphere $= \frac{R}{3}$

$$\Rightarrow \frac{6 \times 10^{-3} m}{3}$$

$$\Rightarrow Bi.No = \frac{5 \times 6 \times 10^{-3}}{20} = 1.5 \times 10^{-3}$$

$$\Rightarrow Bi.No = 1.5 \times 10^{-3} << 0.1$$

$\Rightarrow$ Lumped parameter analysis is valid

$\therefore$ Internal temperature gradients can be neglected.

Ans (b)

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S-2000 ✓

Steel balls 12mm diameter is annealed by heating to 800°c and then slowly cooling to 127°c in air at 50°c. The heat transfer Coefficient for air is 20 W/m²K. Calculate the time reqd for cooling process. The properties of steel are taken as

$K = 45$ W/mK, $\rho = 7830$ kg/m³ $\rho C_p = 600$ J/kg-K

Ans.

$$\text{Biot no} = \frac{20 \times 6\times10^{-3}}{45} = 2.67\times10^{-3} < 0.1$$

⇒ Lumped parameter analysis is valid

$$\Rightarrow \frac{T_i - T_\infty}{T - T_\infty} = e^{\left(\frac{hA}{\rho V C_p}\right)\times \tau}$$

$$\Rightarrow \frac{800-50}{127-50} = e^{\left[\frac{20 \times 4\pi\times(6\times10^{-3})^2}{7830\times\frac{4}{3}\pi(6\times10^{-3})^3\times600}\right]t}$$

$$\Rightarrow 9.74 = e^{\cancel{6.687\times10^{-3}\times t}\ 2.128\times10^{-3}t}$$

$$\Rightarrow \overset{2.128}{\cancel{6.687}}\times10^{-3}t = \ln 9.47$$

$$\Rightarrow t = \cancel{336.2}\ 1056.45$$

$$\Rightarrow \tau = 17.6 \text{ min Ans.}$$

E.S-2008

A Copper sphere Weighing 3 kg is heated in a furnace to a temperature of 300°c and is suddenly taken out and allowed to cool in ambient air at 25°c. If it takes 60 min for the copper sphere to cool down to 35°c what is the average surface heat transfer coefficient?

$\rho_{cu} = 8950$ kg/m³, $C_p = 0.383\ \frac{kJ}{kg°c}$, $K_c = 390$ W/mK

State the Assumptions,

Ans. Let us assume Biot No. is $< 0.1$ i.e. Lumped heat Capacity analysis is valid

$$\Rightarrow \frac{T_i - T_\infty}{T - T_\infty} = e^{\left(\frac{hA}{\rho V C_p}\right)t}$$

$$\Rightarrow \frac{300 - 25}{35 - 25} = e^{\left(\frac{h \times A}{m \times 383}\right)t}$$

$$\rho \times V = m$$

$$\Rightarrow \frac{4}{3}\pi R^3 = \frac{3}{8950}$$

$$\Rightarrow R = 43.1 mm$$
$$= 0.043 m$$

$$\Rightarrow \frac{275}{10} = e^{\left(\frac{h \times 4\pi(0.043)^2}{3 \times 383}\right) \times 3600}$$

$$\Rightarrow 3.314 = h \times 0.0728$$
$$\Rightarrow h = 45.52 W/m^2K$$

now, Calculating Biot no.

$$\Rightarrow Bi. no = \frac{45.52 \times}{390}\left(\frac{0.043}{3}\right) = 1.673 \times 10^{-3} < 0.1$$

So, Our analysis is Correct
and $h = 45.52 W/m^2K$ Ans.

GATE-2005 A small Copper ball of 5mm diameter at 500 K is dropped into an oil bath whose temp. is 300K. The thermal Conductivity of Copper is 400 W/mK its density is 9000 kg/m³ and its specific heat 385 J/kg-K. If the heat transfer Coeff. is 250 W/m²K and lumped analysis is assumed to be valid. The rate of fall of temp of ball at the beginning of Cooling will be in K/s is

(a). 8.7  (b) 13.9  (c) 17.9  (d) 27.7

Ans
$$-\frac{hA}{\rho V C_p}(T - T_\infty) = \frac{dT}{d\tau}$$

$$\Rightarrow -\frac{250 \times 4\pi(0.005)^2}{9000 \times 4 \times \frac{\pi}{3} \times (0.005)^3} \times (500 - 300) = \frac{dT}{d\tau}$$

Writing Energy balance eqn at the instant of $\tau = 0$ sec is

$$\Rightarrow hA(T_i - T_\infty) = -mc_p\left(\frac{dT}{d\tau}\right)$$

$$\Rightarrow h \times \cancel{4\pi} R^{\cancel{2}}(T_i - T_\infty) = -\rho \times \frac{\cancel{4}}{3}\cancel{\pi} R^{\cancel{3}} \times C_p \times \frac{dT}{d\tau}$$

$$\Rightarrow -\frac{(500-300) \times 250 \times 3}{9000 \times 0.0025 \times 385} = \frac{dT}{d\tau}$$

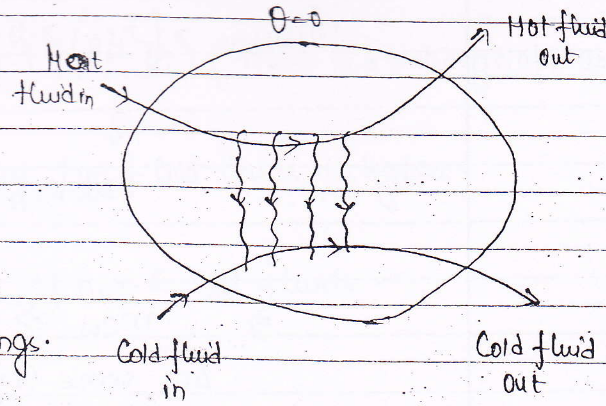
$$\Rightarrow \frac{dT}{d\tau} = -17.3 \text{ K/s}$$

i.e. the rate of Cooling = 17.3 K/s

Rate of Cooling decreases with time.

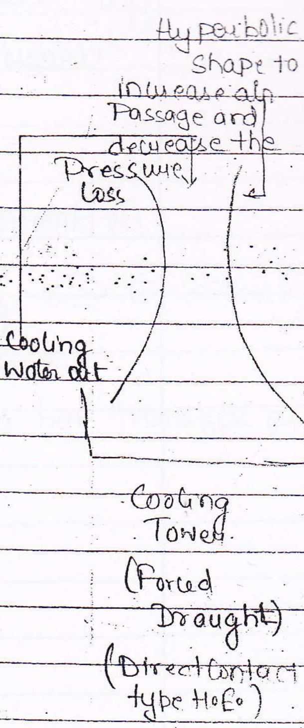
# Heat Exchangers

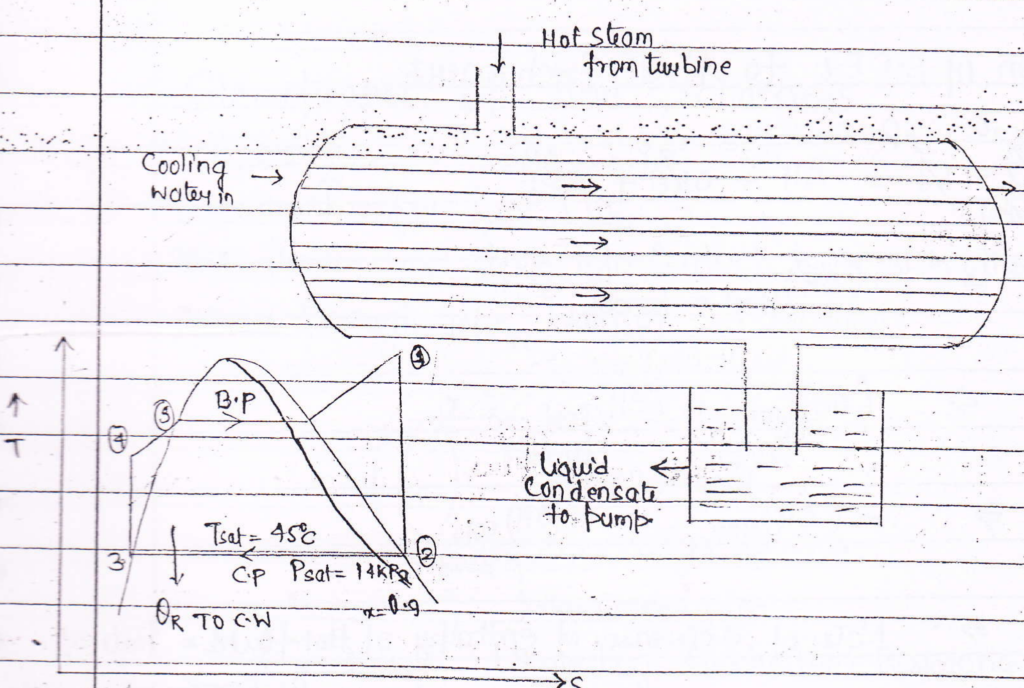
Heat exchanger is a steady flow adiabatic open system in which two flowing streams of fluid exchanged or transferred heat between themselves without losing or gaining heat from the surroundings.




Eg:- o Steam Condenser (Vapour - Liquid)
o Economiser (flue gas - Water)
o Air Preheater (gas to gas) (very low heat transfer rate)
o Cooling Tower (Water - air)
o Jet Condensers (vapour to liquid)
o Oil Coolers. (Liquid to liquid)

## Design of Heat Exchangers




The lowest temp. to which water can be cooled in any cooling tower is Wet Bulb temp of incoming air.

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By Energy balance eqⁿ

$$\dot{m}_{steam} \times (h_{fg}) \times 0.9 = \dot{m}_{cw} \times C_{pw} \times \underbrace{(T_{w_{exit}} - T_{w_{enter}})}_{10^\circ C}$$

$h_{fg} \to 2300\ kJ/kg$, $C_{pw} \to 4.18$

$$\Rightarrow \dot{m}_{cw} >> \dot{m}_{steam}$$

so more cooling is reqd.

Generally 65 cm of mercury is there in Condenser (or 14 kPa)

A proper heat transferred rate must be ensured b/w steam coming from the turbine and the cooling water circulated in the tubes, so that a high vaccum or low absolute pressure of steam can be obtained in the condenser. Otherwise, the vaccum gets reduced rising the pressure inside the condenser thereby decreasing the overall efficiency of the plant.

## Application of S.F.E.E to Heat Exchangers

$$\cancel{Q}^{0} - \cancel{W}^{0} = \Delta H + \Delta \cancel{KE}^{0} + \Delta \cancel{PE}^{0}$$
(adiabatic)

$$\Rightarrow (\Delta H)_{\text{Heat exchanger}} = 0$$

$$\Rightarrow (\Delta H)_{\text{Hot fluid}} + (\Delta H)_{\text{cold fluid}} = 0$$

$$\Rightarrow -(\Delta H)_{\text{Hot fluid}} = +(\Delta H)_{\text{cold fluid}}$$

$\Rightarrow$ Rate of decrease of enthalpy of Hot fluid = Rate of

In Heat Transfer we deals with energy transfer.

as pressure is constant since the length of heat exchanger is small

Enthalpy increase of Cold fluid.

$$\Rightarrow \dot{m}_h C_{ph} (T_{hi} - T_{he}) = \dot{m}_c C_{pc} (T_{ce} - T_{ci})$$ (there is no change of phase)

$\dot{m}_h$ & $\dot{m}_c$ are mass flow rates of Hot & Cold fluids in kg/sec.

$C_{ph}$ & $C_{pc}$ are specific heats of hot & cold fluids.

$T_{hi}$ → Hot fluid inlet temp.
$T_{he}$ → Hot fluid exit temp.
$T_{ci}$ → Cold fluid inlet temp.
$T_{ce}$ → Cold fluid exit temp.

It is Commonly assumed in any heat exchanger that neither hot fluid nor cold fluid loose any pressure while passing through the heat exchanger.

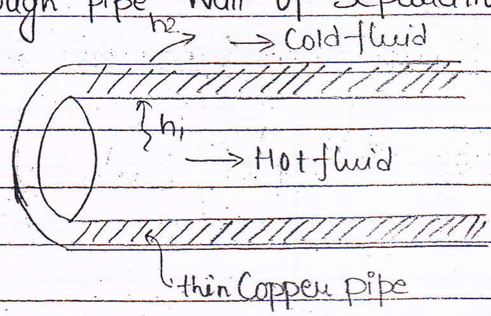
In case of phase change

$$Q = \dot{m}_{steam} (\Delta h) \text{ Watts}$$

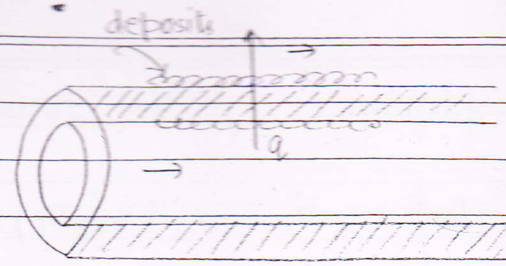
## Classification of Heat Exchangers

0 Direct Transfer type HE.

Both fluids donot come into Contact with each other but transfer of heat occurs through pipe wall of Separation.




$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}$$ (Neglecting Conduction Resistance & without fouling)




Fouling

Fouling factor is the factor which takes into account any thermal resistance offered by scaling or any deposits formed due to chemical rxn b/w the fluid and pipe material also due to accumulation of suspended particles.
Fouling has units of ( ~~W/m~~ $m^2K/W$ ).

With fouling on both sides.

$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2} + f_1 + f_2$$

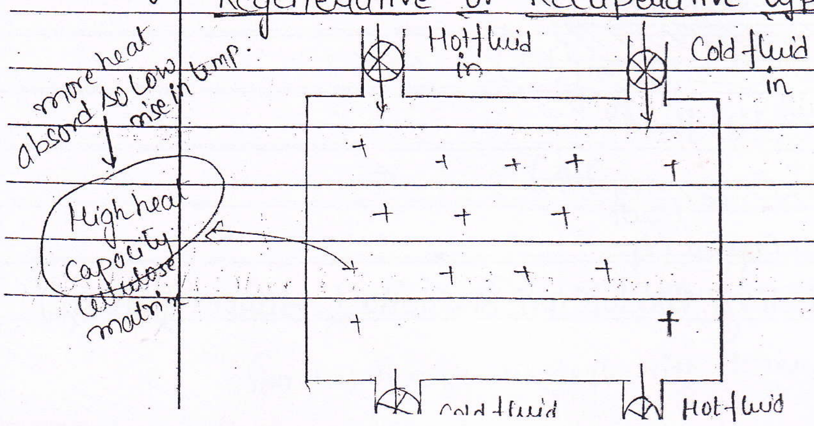
$f_1$ & $f_2$ are fouling factors of hot side & cold side resp.

Eg- Surface condenser, economiser

0 Direct Contact type Heat Exchanger
Both the fluids physically mix with each other at the same pressure.

Eg- Jet Condenser (Steam & Water) & Cooling Tower (Water & atm. air)

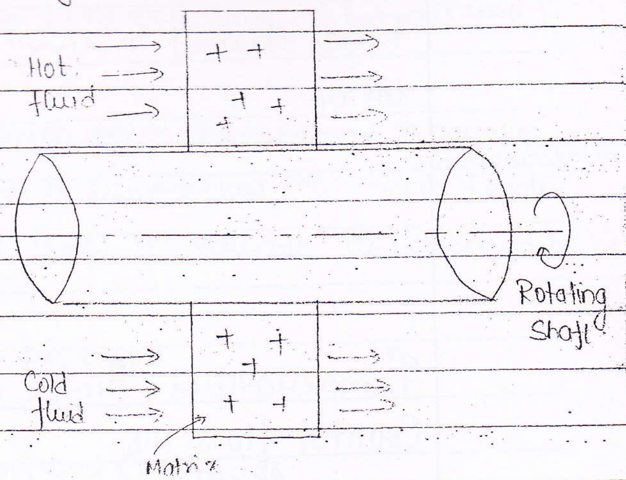
0 Regenerative or Recuperative type Heat Exchanger



Both the fluids are alternatively passing through the Matrix of H.E. One heating it and the other picking on heat.

Eg- LJungstorm air preheater (Gas turbines)

but there is chances of leakage.

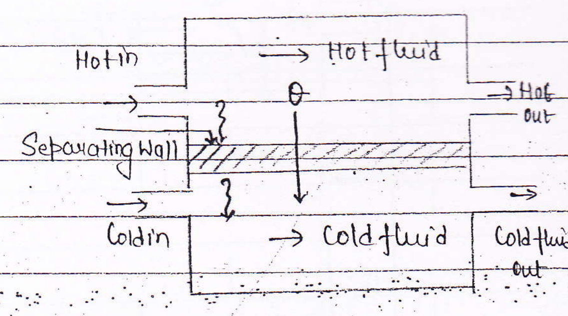



Rotating Matrix type of Regenerative H.E.

## Classification of Direct Transfer Type Heat Exchanger

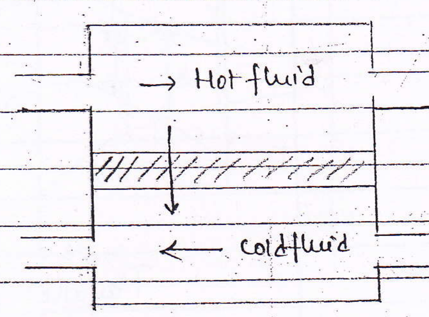
o Parallel Flow Heat Exchanger.

Fluid flows in same dirxn

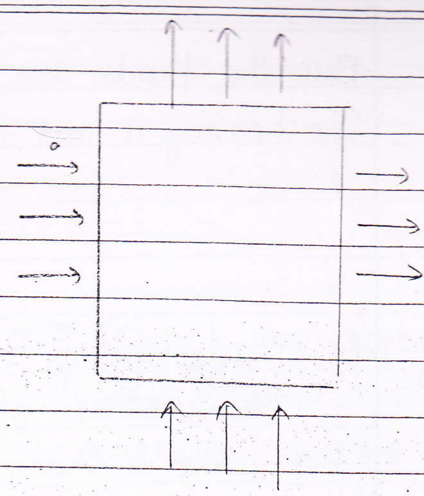



o Counter Flow Heat Exchanger

Fluid flows in opposite dirxn.

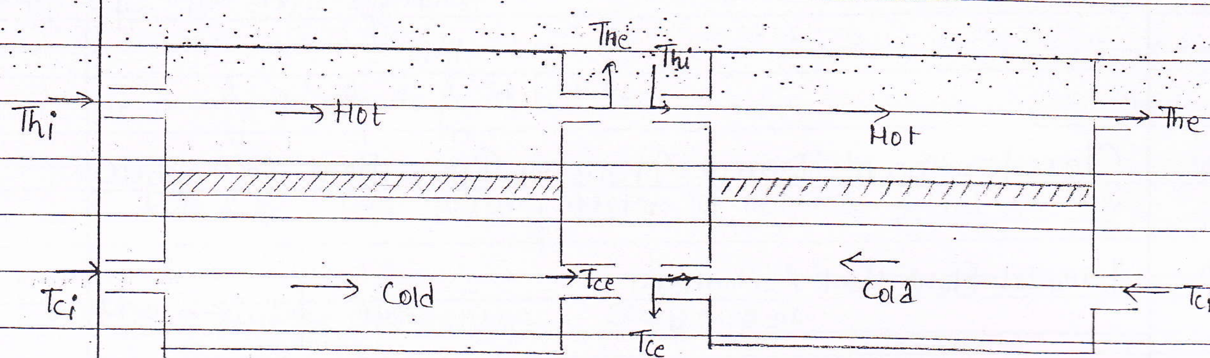


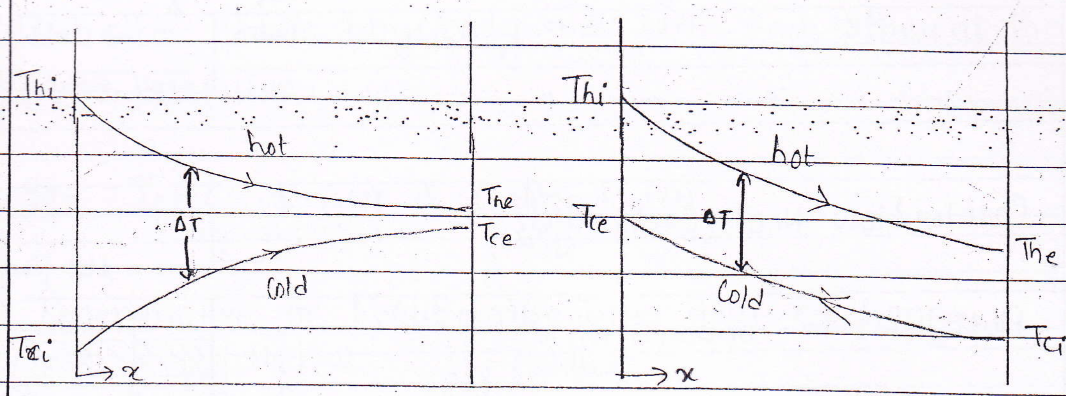
## Cross Flow Heat Exchanger



Fluids travel in $dirx^n$ $\perp^r$ to each other.

## Temperature Profile of Hot & Cold fluid in parallel and Counter flow H.E.




Parallel flow

Counterflow

Counterflow is less irreversible than parallel flow. In counterflow

$T_{ce}$ ~~gen~~ can be greater than $T_{he}$.

Note :- $T_{ce}$ may be greater than $T_{he}$ (Cold exit temp > hot exit temp) in Counter flow.

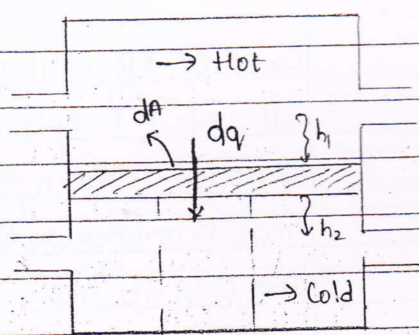
- The variation of $\Delta T$ w.r.t $x$ is much more pronounced in parallel flow heat exchanger as compared to Counterflow H.E., hence, heat transfer in Counter flow H.E. is thermodynamically lesser irreversible as compared to that in parallel flow H.E.
- For same heat exchanger area provided,

$$Q_{counter} > Q_{parallel}$$

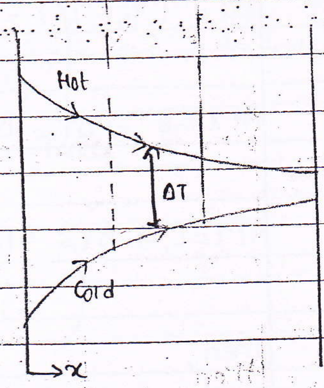
$Q$ = Heat transfer rate b/w hot and cold fluids.

## Mean Temperature Difference ($\Delta T_m$)

Consider a differential area of H.E through which differential heat transfer rate b/w hot and cold fluids is 'dq' (not constant)




$$\frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2}$$




$$\therefore \; dq = U \Delta T \, dA$$

$$\int_{inlet}^{exit} dq = \int_{inlet}^{exit} U \Delta T \, dA$$

$\Rightarrow Q$ = Total heat transfer rate

$$= U \int_{inlet}^{exit} \Delta T \, dA \quad \text{—— (1)}$$

Mean Temperature difference is the parameter which takes into account variation of $\Delta T$ w.r.t $x$ (i.e. along the length of the H.E.) and hence is defined from the eq$^n$

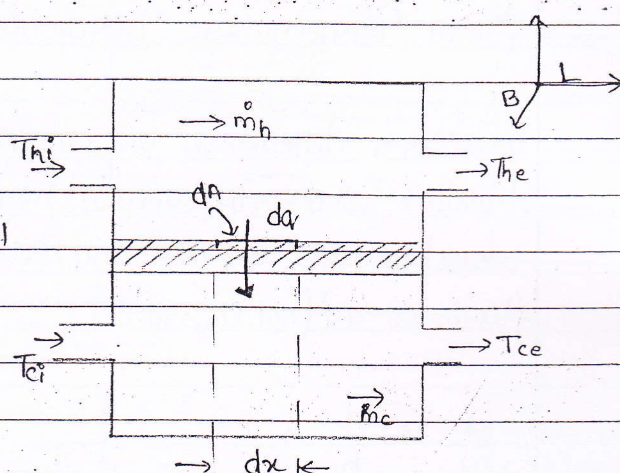
$$\theta = UA\,\Delta T_m \quad \text{—— ②}$$

A → Total Heat transfer area of Heat Exchanger.

Comparing ① and ② we get

$$\Delta T_m = \frac{1}{A}\int_{\text{inlet}}^{\text{exit}} \Delta T\, dA$$

To get the eq$^n$ of LMTD in parallel flow H.E.

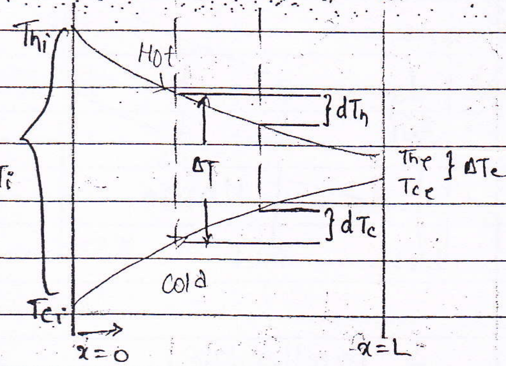



Consider differential area 'dA' of H.E of length 'dx' through which the differential heat transfer rate b/w hot and cold fluids is 'dq'.

$\Delta T = (T_h - T_c) = f(x)$

At $x = 0 \Rightarrow \Delta T = \Delta T_i = T_{hi} - T_{ci}$

At $x = L, \Rightarrow \Delta T = \Delta T_e = T_{he} - T_{ce}$




then,

$$dq = U\Delta T\, dA = U\Delta T\, B\, dx$$

$$dA = B\, dx$$

Where B = Width of Separating Wall.

$$dq = -\dot{m}_h C_{ph}\, dT_h$$
$$= +\dot{m}_c C_{pc}\, dT_c$$

$$\Delta T = T_h - T_c$$

$$\Rightarrow d(\Delta T) = dT_h - dT_c$$

$$= \frac{-dq}{\dot{m}_h C_{ph}} - \frac{dq}{\dot{m}_c C_{pc}}$$

$$= -\left(\frac{dq}{\dot{m}_h C_{ph}} + \frac{dq}{\dot{m}_c C_{pc}}\right)$$

$$= -dq\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

$$\Rightarrow d(\Delta T) = -U\Delta T\, B\, dx\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

$$\Rightarrow \int_{\Delta T_i}^{\Delta T_e} \frac{d(\Delta T)}{\Delta T} = -\int_{x=0}^{x=+L} UB\, dx\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

$$\Rightarrow \ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UBL\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

$BL = A =$ Total heat exchanger Area,

$$\Rightarrow \ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left(\frac{1}{\dot{m}_h C_{ph}} + \frac{1}{\dot{m}_c C_{pc}}\right)$$

But we know that

Rate of H.T $(\theta) = \dot{m}_h C_{ph}(T_{hi} - T_{he})$

$= \dot{m}_c C_{pc}(T_{ce} - T_{ci})$

$$\Rightarrow \ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left[\frac{T_{hi} - T_{he}}{\theta} + \frac{T_{ce} - T_{ci}}{\theta}\right]$$

$$= \frac{UA}{\theta}\left[\Delta T_i - \Delta T_e\right]$$

$$\Rightarrow \theta = \frac{UA\left(\Delta T_i - \Delta T_e\right)}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

Comparing with

$$\theta = UA\,\Delta T_m$$

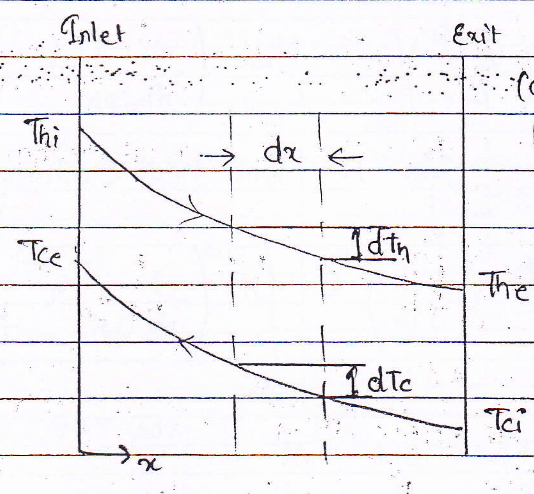
$$\Rightarrow (\Delta T_m)_{parallel\,flow} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

Hence, the Variation of $\Delta T$ w.r.t $x$ in parallel flow H.E. is Logarithmic from inlet to exit.

here in Counterflow $\Delta T_i$ will always be greater than $\Delta T_e$.

3/12/2011

Counterflow.




(as reference fluid is hot fluid)

hot fluid entering on left and exiting on right.

$$\Delta T = f(x)$$

At $x = 0 \Rightarrow \Delta T = \Delta T_i = T_{hi} - T_{ce}$

At $x = L \Rightarrow \Delta T = \Delta T_e = T_{he} - T_{ci}$

$$dq = UB\,dx\,\Delta T$$
$$= -\dot{m}_h C_{ph}\, dT_h$$
$$= -\dot{m}_c C_{pc}\, dT_c$$

{Both -ve as in +ve dirⁿ of $x$ both fluids temp. are decreasing}

$$\Delta T = T_h - T_c$$
$$d(\Delta T) = dT_h - dT_c$$
$$= -\frac{dq}{\dot{m}_h C_{ph}} + \frac{dq}{\dot{m}_c C_{pc}}$$
$$= -dq\left(\frac{1}{\dot{m}_h C_{ph}} - \frac{1}{\dot{m}_c C_{pc}}\right)$$
$$= -UB\,dx\,\Delta T\left(\frac{1}{\dot{m}_h C_{ph}} - \frac{1}{\dot{m}_c C_{pc}}\right)$$
$$\Rightarrow \frac{d(\Delta T)}{\Delta T} = -UB\,dx\left(\frac{1}{\dot{m}_h C_{ph}} - \frac{1}{\dot{m}_c C_{pc}}\right)$$

Integrating both sides,

$$\int_{\Delta T_i}^{\Delta T_e} \frac{d(\Delta T)}{\Delta T} = -UB\int_0^L dx\left(\frac{1}{\dot{m}_h C_{ph}} - \frac{1}{\dot{m}_c C_{pc}}\right)$$

$$\Rightarrow \ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UBL\left[\frac{1}{\dot{m}_h C_{ph}} - \frac{1}{\dot{m}_c C_{pc}}\right]$$

But $BL$ = Total Heat Transfer area of H.E.

$$Q = \dot{m}_h C_{ph}(T_{hi} - T_{he}) = \dot{m}_c C_{pc}(T_{ce} - T_{ci})$$

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$$\Rightarrow \ln\left(\frac{\Delta T_i}{\Delta T_e}\right) = UA\left[\frac{T_{hi}-T_{he}}{\theta} - \frac{T_{ce}-T_{ci}}{\theta}\right]$$

$$= \frac{UA}{\theta}\left[\Delta T_i - \Delta T_e\right]$$

$$\Rightarrow \theta = UA\left[\frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}\right]$$

Comparing with $\theta = UA\Delta T_m$

$$\Rightarrow \Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)} \quad [\text{L.M.T.D for Counter-flow}]$$

Note: ◦ Even though the formula for LMTD is same in both parallel and Counter flow heat exchangers, the definitions of $\Delta T_i$ and $\Delta T_e$ are different b/w them.
◦ For the same inlet and exit temperatures of both hot and cold fluids the value of LMTD is greater for Counter-flow than in parallel flow.

I.A.T.E-2005 : Hot Oil is Cooled from 80°c to 50°c in a oil Cooler which ~~too~~ uses air as the Coolant, the air temperature rises from 30-40°c. The designer Uses a LMTD value of 26°c, the type of heat exchanger is

(a). Parallel flow
(b). Double flow
(c). Counter flow
(d). Cross flow

Ans. $L.M.T.D = \dfrac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$

$\Rightarrow \quad 26 = \dfrac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$

$\Delta T_i$ (Parallel) $= \dfrac{(80-30) - (50-40)}{\ln\left(\frac{50}{10}\right)} = 24.8^\circ c \neq 26^\circ c$

$\Delta T_i$ (Counter) $= 28.85^\circ c \neq 26^\circ c$

In double pipe H.E. there is possibility of both parallel and counter flow so it is a doubt which flow is to be used

For Cross-flow

$$\boxed{(\Delta T_m)_{crossflow} = (\Delta T_m)_{counter} \times F}$$

$F \rightarrow$ Correction factor from charts $< 1$

$(\Delta T_m)_{counter} > (\Delta T_m)_{cross} > (\Delta T_m)_{parallel}$

** For the same heat transfer rate req$^d$ in both parallel & counter flow H.E. The area required for counter flow is lesser than that of parallel flow.

## Design of Heat Exchangers

In any design of H.E., the area is unknown, by knowing that no. of tubes and other parameters can be found.

## To find the area of H.E:

A. LMTD Method

Given Data

- Both the mass flow rates of hot and cold fluid ($\dot{m}_h$ & $\dot{m}_c$)
- Both the Specific heats ($C_{ph}$ & $C_{pc}$)

  $C_{p\,air} = 1.005\ kJ/kg\text{-}K$ , $C_{pw} = 4.187\ kJ/kg\text{-}K$
- Overall HoT Coefficient $U (W/m^2K)$

  $$\Rightarrow \frac{1}{U} = \frac{1}{h_1} + \frac{1}{h_2} + F_1 + F_2$$
- Only 3 Temperatures known among 4

Solution

Step 1 Calculate $4^{th}$ Unknown Temp. from Energy balance eqn or Heat balance eqn i.e. $\dot{m}_h C_{ph}(T_{hi} - T_{he}) = \dot{m}_c C_{pc}(T_{ce} - T_{ci})$

Step 2 Draw the temperature profiles of Hot and Cold fluid based on what type of H.E. is to be designed.

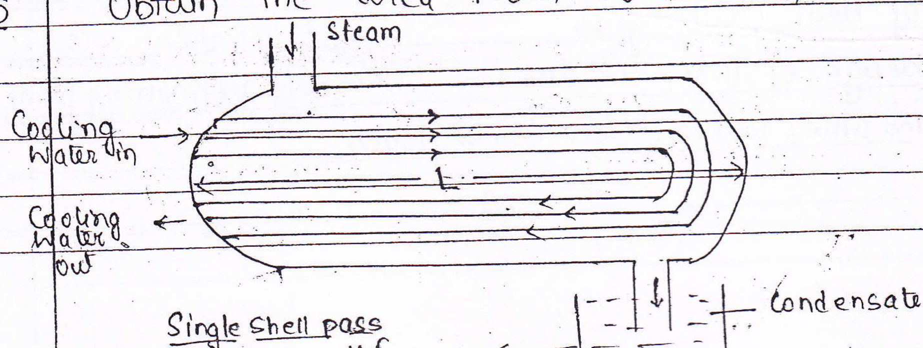
Step 3 Calculate LMTD from

$$\Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i/\Delta T_e)}$$

Step 4 Calculate heat transfer rate b/w hot and cold fluids from

$$Q = \dot{m}_h C_{ph}(T_{hi} - T_{he}) = \dot{m}_c C_{pc}(T_{ce} - T_{ci})$$

Step 5 Obtain the area from $Q = UA\Delta T_m$




Single shell pass 2 Tube pass H.f.

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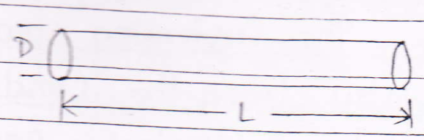



$A = \pi D \times L \times n \times P$

$n \rightarrow$ no. of Tubes per pass (for more heat transfer rate)

$p \rightarrow$ no. of passes (for more heat exchange area)

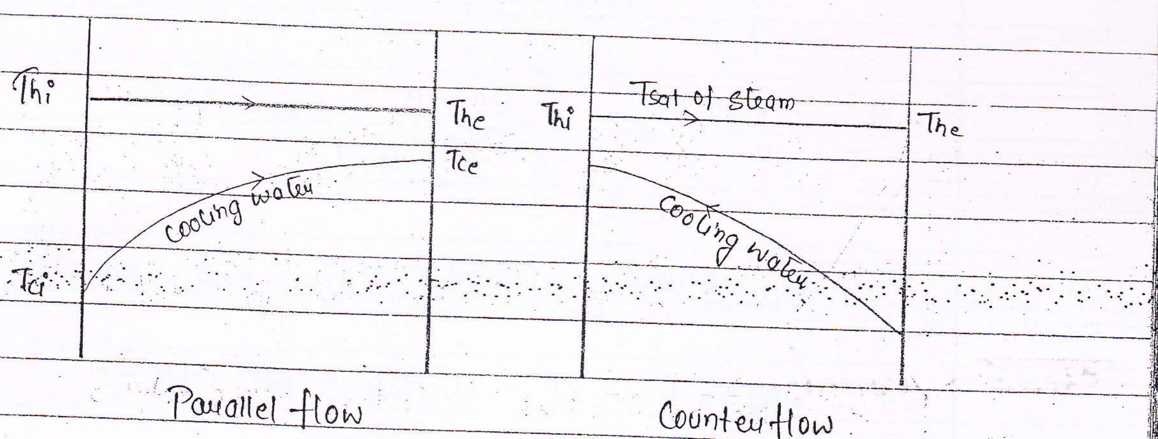
'$n$' can be calculated from continuity eq$^n$

$\Rightarrow \dot{m} = \rho \times \frac{\pi}{4} (D_i)^2 \vec{V} \times n$ kg/sec

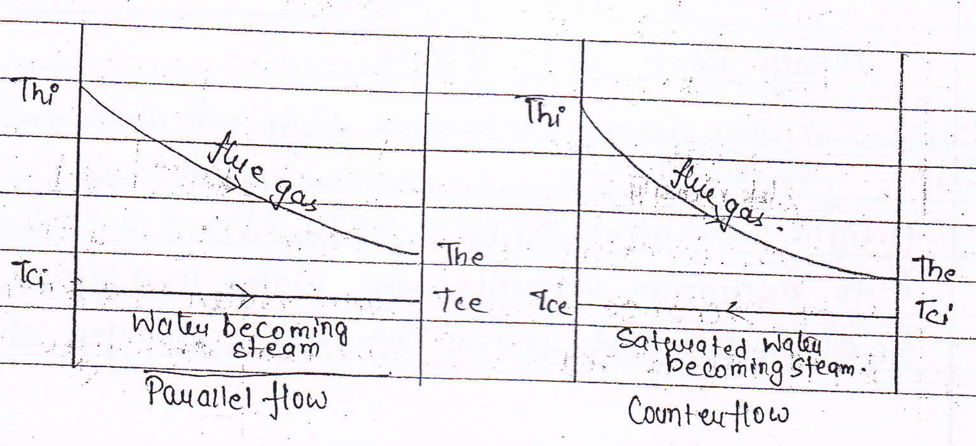
2 Special Cases regarding L.M.T.D

Case(i) When one of the fluids is changing its phase like Condensor or Boiler (Evaporator)

Steam Condenser




Steam Generator




** During Phase change the L.M.T.D for both parallel and Counter i.e. $(\Delta T_m)_{parallel} = (\Delta T_m)_{counter}$

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A.T.E-2004 In a Condenser water enters at 30°c and flows at the rate of 1500 kg/hr. The Condensing steam is at a temp of 120°c and cooling water leaves the Condenser at 80°c. If the overall heat transfer Coefficient is 2000 W/m²k, The heat transfer Area is

(a). 0.707 m²
(b). 7.07 m²
(c). 70.7 m²
(d). 141.4 m²

Ans.

$T_{ci} = 30°c$

$\dot{m}_c = 1500 \text{ kg/hr} = 0.416 \text{ kg/s}$

$T_{hi} = 120°c = T_{he}$

$T_{ce} = 80°c$

$\Rightarrow \Delta T_i = (120-30) = 90°c$

$\Delta T_e = (120-80) = 40°c$

$$\Rightarrow L.M.T.D = \frac{90-40}{\ln(90/40)} = 61.65°c$$

$\Rightarrow A \times 2000 \times \Delta T_m = 0.416 \times 4180 \frac{J}{kg\text{-}k} \times \cancel{6}.(\overset{80-30}{\cancel{120-80}})$

$\Rightarrow 2000 \times 61.65 \times A = 0.416 \times 4180 \times \cancel{40}\ 50$

$\Rightarrow A = 0.705 \text{ m}^2$

Always keep $C_p$ in $\underline{\underline{J/kg\text{-}k}}$.

IES 1990 A light (less viscosity) lubricating oil $C_p = 2090$ J/kg-k is cooled by allowing it to exchange energy with water in a small heat exchanger. The oil enters and leaves the heat exchanger at 375k and

350K and flows at a rate of 0.5 kg/s. Water at 280 K is available in sufficient quantity to allow 0.201 kg/s to be used for cooling purposes. Determine the required heat transfer area for Counter flow H.E. $U = 250\ W/m^2K$.

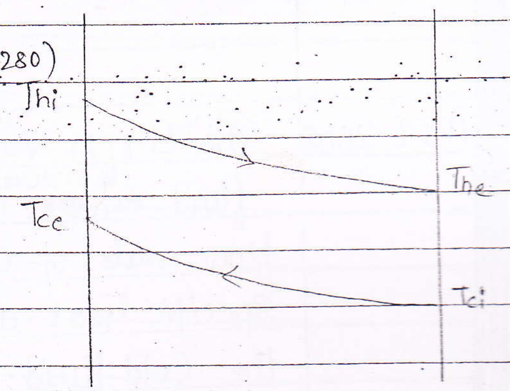
Ans- $\dot{m}_o = 0.5\ kg/s$ $\quad$ $\dot{m}_w = 0.201\ kg/s$

$C_{po} = 2090\ J/kg\text{-}K$ $\quad$ $C_{pw} = 4200\ J/kg\text{-}K$

$T_{hi} = 375\ K$ $\quad$ $T_{hi} = 280\ K$

$T_{he} = 350\ K$ $\quad$ $T_{ce} = ?$

$\Rightarrow 0.5 \times 2090 \times (375 - 350) = 0.201 \times 4200 \times (T_{ce} - 280)$




$\Rightarrow T_{ce} = 311\ K$

$$\Delta T_m = \frac{(375-311) - (350-280)}{\ln\left(\frac{(375-311)}{(350-280)}\right)}$$

$$= \frac{64 - 70}{\ln(64/70)} = 66.955$$

$\Rightarrow Q = 0.5 \times 2090 \times (375 - 350) = 250 \times A \times 66.95$

$\Rightarrow A = \underline{1.56\ m^2}$ Ans.

Case (ii) When both the fluids have equal capacity rate in Counter flow H.E.

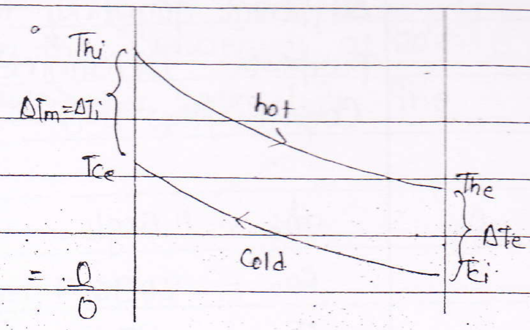
i.e. $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$

Then, by heat balance Eqn,

$$\dot{m}_h C_{ph}(T_{hi} - T_{he}) = \dot{m}_c C_{pc}(T_{ce} - T_{ci})$$

$$\Rightarrow T_{hi} - T_{he} = T_{ce} - T_{ci}$$

$$\Rightarrow T_{hi} - T_{ce} = T_{he} - T_{ci}$$




$$\Rightarrow \Delta T_i = \Delta T_e$$

$$\Rightarrow \Delta T_m = \frac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i/\Delta T_e)} = \frac{0}{0}$$

from L'Hospitals rule:

$$(\Delta T_m)_{counter} = \Delta T_i \text{ or } \Delta T_e$$

A.T.E-2008 The LMTD value of a Counter flow H.E. is 20°C, the cold fluid ~~enters~~ enters at 20°C and the hot fluid enters at 100°C. Mass flow rate of cold fluid is twice that of the hot fluid. Specific heat at constant pressure of hold is twice that of the cold fluid. The exit temperature of the cold fluid is

(a). 40°C

(b) 60°C

(c) 80°C

(d). cannot be determined.

Ans.

$$\cancel{\dot{m}_h c_{ph}}(T_{hi} - T_{he}) = \cancel{\dot{m}_c c_{pc}}(T_{ce} - T_{ci})$$

$$\Rightarrow \Delta T_i = 20 = (T_{hi} - T_{ce})$$

$$\Rightarrow 100 - T_{ce} = 20$$

$$\Rightarrow T_{ce} = \underline{80^\circ C} \text{ Ans.}$$

I.E.S-2009 Find the surface area required by a surface condenser dealing with 25000 kg of saturated steam per hour at a pressure of 0.5 bar. Temperature of condensing water is 25°c. Cooling water is heated from ~~50°c~~ 15°c to 25°c while passing through condenser. Assume a heat transfer coefficient of 10 kW/m²K, the condenser has 2 water passes with tubes of 19 mm outer diameter and 1.2 mm thickness. Find the length and no. of tubes per pass. Assume velocity of water is 1m/s. Assume correction factor for two tube pass exchanger as 0.86. At 0.5 bar the saturation temp. is 32.55°C and the latent heat is 2560 kJ/kg. $\rho_w = 1000 kg/m^3$, $C_{pw} = 4.2 kJ/kg\text{-}K$.

Ans.

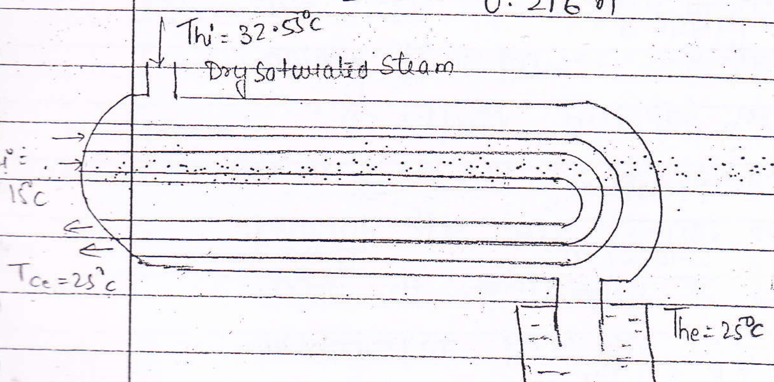
$$\dot{m}_s = \frac{25000 kg}{3600} = 6.94 kg/sec$$

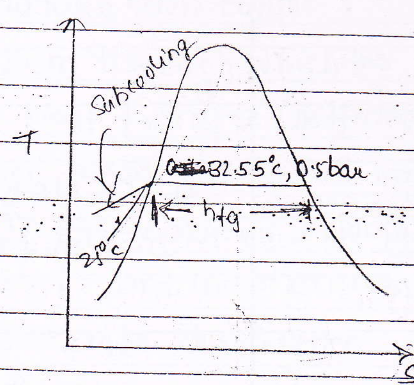
$$\dot{m}_w = \rho \times \frac{\pi}{4}(D_i)^2 \times \vec{V} \times n$$

$$D_i = 19mm - 1.2mm \times 2$$
$$= 16.6mm$$

$$= 1000 \times \frac{\pi}{4} \times (16.6)^2 \times 10^{-6} \times 1 \times n$$

$$= 0.216 n$$




Since there is a temp. drop below saturation temp. ∴ there will be subcooling and heat rejected by steam = L.H + sensible heat.

$$(\Delta T_m)_{counter} = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)}$$

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$T_{hi} = 32.55°c$

$T_{he} = 25°c$

$T_{ci} = 15°c$

$T_{ce} = 25°c$

$\Rightarrow \Delta T_i = 32.55 - 25 = 7.55°c$

$\Delta T_e = 25 - 15 = 10°c$

$\Rightarrow (\Delta T_m)_{Counter} = \dfrac{\Delta T_i - \Delta T_e}{\ln(\Delta T_i / \Delta T_e)} = \dfrac{7.55 - 10}{\ln(7.55/10)} = 8.71°c$

$\Rightarrow (\Delta T_m)_{Cross\ or\ Condenser} = 8.71 \times 0.86 = 7.49°c$

$Q$ = Rate of Heat Transfer from steam to water

$= \dot{m}_{steam} \, (\Delta h)_{steam}$

$= \dfrac{25000}{3600} \times \left[ h_{fg} + (C_{pw})(\Delta T)_{subcooling} \right]$

$= \dfrac{25000}{3600} \times \left[ 2560 + 4.18 \times (32.55 - 25) \right]$

$= 17996.9$ kW.

$\Rightarrow U A \Delta T_m = Q$

$\Rightarrow 10 \times A \times 7.49 = 17996.9$

$\Rightarrow A = 240.27 \ m^2$ Ans.

$\Rightarrow \pi \times D_o \times L \times n \times p = A$

$\Rightarrow \pi \times 19 mm \times L \times n \times 2 = 240.27 \times 10^6$

$\Rightarrow L \times n = 2.0126 \times 10^6$

$\Rightarrow L \times n = 2012.6$ m

By Energy balance eqn.

$$\rho \times \frac{\pi}{4}(D_i)^2 \times \vec{V} \times n \times 4.2 \times (25-15) = 17996.9$$

$$\Rightarrow 1000 \times \frac{\pi}{4} \times (16.6 \times 10^{-3})^2 \times 1 \times n \times 4.2 \times 10 = 17996.9$$

$$\Rightarrow n = \underline{\underline{1980}} \text{ Ans}$$

$$\Rightarrow L \times n = 2012 \quad \Rightarrow L \times 1980 = 2012$$

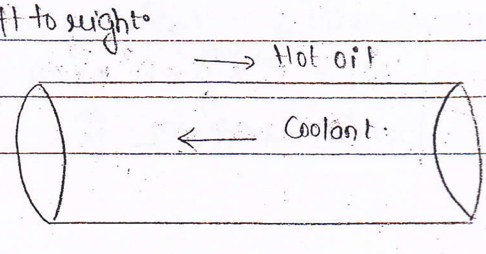
$$\Rightarrow L = \underline{1.016 \text{ m}} \text{ Ans}$$

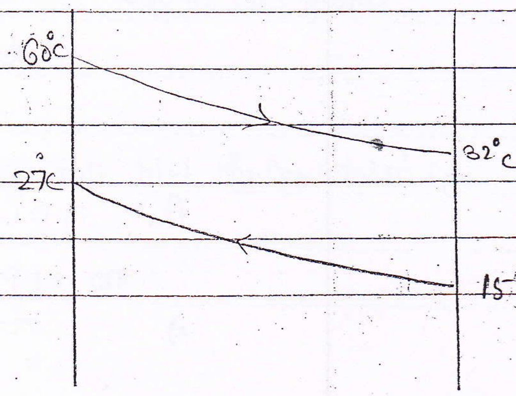
IES-2010. A Counter flow heat exchanger is to be designed to cool 900kg/hr of oil from 60°c to 32°c using a fluid with specific heat 1kJ/kg-K at 15°c the specific heat of oil is 0.5kJ/kg-K and the maximum allowable exit temp. of the cooling fluid is 27°c. Work out the following

(i). Sketch the System and show the temperature distribution.

(ii). Find NTU if diameter of the tube is 2cm through which the cooling fluid passes and the overall heat transfer coefficient is 200 W/m²K, find the no. of tubes reqd and the tube length. Assume $\rho_{\text{cooling fluid}}$ = 250 kg/m³. If the maximum velocity through the tube cannot exceed 2m/s and the maximum length of exchanger is limited to 12m due to space restriction. Find the Configuration (arrangement) of H.E. and sketch the final design.

Ans. General notion is keep flow of hot fluid from left to right.



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$\Delta T_i = 33°C$

$\Delta T_e = 17°C$

$$\Rightarrow \Delta T_m = \frac{33-17}{\ln\left(\frac{33}{17}\right)} = \cancel{24.12}\ 24.12°C$$

$U = 200\ W/m^2K$

$$\Rightarrow \theta = UA\Delta T_m = \dot{m}_h C_{ph}\left(T_{hi} - T_{he}\right) \text{ Watts}$$

$$\Rightarrow 200 \times A \times 24.12 = \cancel{\rho \times \frac{\pi}{4} \times (D_i)^2 \times \vec{V} \times 1 \times (27-15) \times 1000}$$

$$= \cancel{250 \times 0.7854 \times (0.02)^2 \times 12^2 \times 12 \times 1000}$$

$$\Rightarrow A = \frac{\frac{900}{3600} \times 500 \times (60-32)}{200 \times 24.12}$$

$$\Rightarrow A = 0.725\ m^2$$

$\Rightarrow$ ~~$A = \pi D L \times n \times p$~~

$$\frac{\dot{m}_h c_{ph}}{\dot{m}_c c_{pc}} < 1 \Rightarrow \frac{125}{242.64} < 1 = 0.51$$

(ii). $$NTU = \frac{UA}{\dot{m} c_p} = 1.16$$

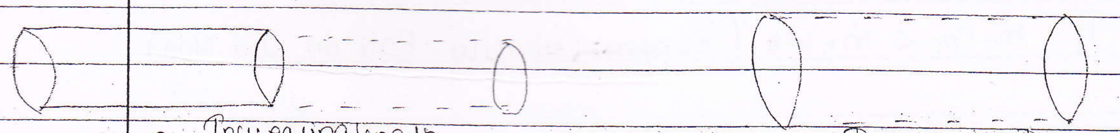
(iii) $$\dot{m}_c = \frac{\theta}{C_{pc}(T_{ce} - T_{ci})} = \frac{\dot{m}_h C_{ph}(T_{hi} - T_{he})}{C_{pc} \times (T_{ce} - T_{ci})}$$

$$= 0.2916\ kg/sec$$

| | |
|---|---|
| $A = \pi D L \times n \times p$ | $\dot{m}_c = \rho \times \frac{\pi}{4} \times D^2 \times \vec{V} \times n$ |
| no. of Pass $= 1$ | $n = 1.39 \approx 2$ |
| $\Rightarrow n = 2$, $L = 5.67\ m$ | |

When no. of pass is 2.

$\Rightarrow \dot{m}_c = \frac{\rho \times \pi D^2}{4} \times \vec{V} \times 2$ $\Rightarrow \vec{V} = 3.712 m/s > 2 m/s$.




If a single tube is provided having a same heat exchanger area and same diameter then the length of the tube becomes $2 \times 5.67m$ which is lesser than available space of 12m but then the velocity of coolant becomes 3.7 m/s which is more than the max. allowable velocity of 2m/s hence final design may be 2 tubes.

The other option available is increase the diameter of the tube thereby bringing the velocity down to 2m/s and hence provide a single tube.

i.e. The single tube with 2 pass is replaced by 2 tubes with single pass.

## Effectiveness of H.E.

It is defined as ratio b/w actual heat transfer rate that is taking place b/w hot and cold fluid and the maximum possible heat transfer rate b/w them.

$$\epsilon_{HE} = \frac{q_{act}}{q_{max.possible}}$$

$q_{act}$ = Actual H.T. rate $= \dot{m}_h C_{ph}(T_{hi} - T_{he}) = \dot{m}_c C_{pc}(T_{ce} - T_{ci})$

$q_{max}$ = Maximum possible H.T rate b/w hot and cold fluids

$= (\dot{m} C_p)_{smaller} \times (T_{hi} - T_{ci})$

Where $(\dot{m} c_p)_{small}$ is the smaller capacity rate b/w $\dot{m}_h C_{ph}$ and $\dot{m}_c C_{pc}$

If $\dot{m}_h C_{ph} < \dot{m}_c C_{pc}$ then,

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$$\epsilon_{HE} = \frac{\dot{m}_h C_{ph}(T_{hi} - T_{he})}{\dot{m}_h C_{ph}(T_{hi} - T_{ci})} = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$$

If $\dot{m}_c C_{pc} < \dot{m}_h C_{ph}$ (Smaller Capacity Rate on Cold Side)

$$\epsilon_{HE} = \frac{\dot{m}_c C_{pc}(T_{ce} - T_{ci})}{\dot{m}_c C_{pc}(T_{hi} - T_{ci})} = \left(\frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}\right)$$

If $\dot{m}_h C_{ph} = \dot{m}_c C_{pc}$, Then only.

$$\epsilon = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}} = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$

They remain applicable to any type of H.E.

Usually, $\boxed{0.65 < \epsilon < 0.75}$

Capacity Rate Ratio

$$C = \frac{(\dot{m}C_p)_{smaller}}{(\dot{m}C_p)_{big}} \qquad 0 \leq C \leq 1$$

(During Phase Change $C_p$ is $\infty$, for $C = 0$.
$Q = \dot{m}C_p \Delta T$ as $\Delta T = 0$ for phase Change)

** C value becomes '0' only when one of the fluids undergo phase change like in Condenser or boiler (evaporator)

Number of Transfer Units (NTU)

$$NTU = \frac{UA}{(\dot{m}C_p)_{smaller}}$$

$U$ ← W/m²K

$(\dot{m}C_p)$ ← J/kg-K

NTU being directly proportional to the Area of the heat exchanger it indicates the size of the heat exchanger. Also if NTU is higher, rate of H.T will be more.

For any Heat Exchanger,

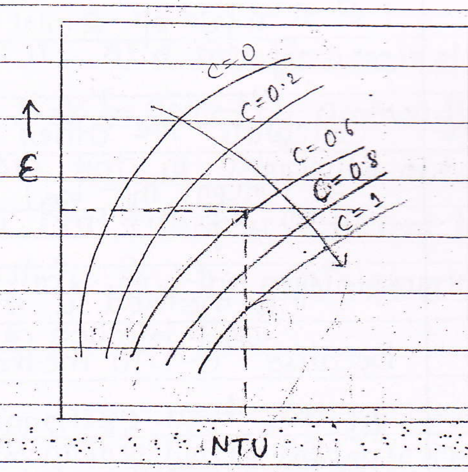
$$\epsilon = f(NTU, C)$$

$$\boxed{\epsilon_{\text{parallel flow}} = \frac{1 - e^{-(1+C)NTU}}{1+C}}$$

$$\boxed{\epsilon_{\text{counter flow}} = \frac{1 - e^{-(1-C)NTU}}{1 - C\,e^{-(1-C)NTU}}}$$

## $\epsilon$ - NTU Method

This method is particularly useful when both the exit temperatures of hot and cold ($T_{he}$ & $T_{ce}$) are to be calculated for a given H.E. Area




Given Datas

- Both the mass flow rates of hot and cold fluids ($\dot{m}_h$ & $\dot{m}_c$)
- Both specific heats $C_p$ & $C_h$
- Overall H.T. Coefficient (U)
- Only Two inlet Temperatures ($T_{hi}$ & $T_{ci}$)
- Area (A) of H.E.

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Solution

Step 1. Calculate both the capacity rates of hot and Cold fluids i.e. $\dot{m}_h C_{ph}$ & $\dot{m}_c C_{pc}$

Step 2 Obtain $C = \frac{(\dot{m} C_p)_s}{(\dot{m} C_p)_b}$

Step 3 Calculate $NTU = \frac{UA}{(\dot{m} C_p)_{small}}$ ← W/m²K, ← J/kg-K

Step 4 Calculate effectiveness of the H.E. ~~based on which side has~~ ~~sm~~ Since it is a fxⁿ of "C & NTU"

Step 5 Calculate only one exit temperature of the fluids based on which side has smaller capacity rate.

$\epsilon$ is a fxⁿ of 2 inlet and one exit temp. $= \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}}$ or $\frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$

Step 6 Calculate the other exit temp. from Energy balance eqⁿ.
$\Rightarrow \dot{m}_h C_{ph}(T_{hi} - T_{ce}) = \dot{m}_c C_{pc}(T_{ce} - T_{ci})$

** This method is more versatile than LMTD method because $\epsilon$-NTU method can also be used to design the area of the heat exchanger by using the same data given in LMTD method.

Special Case (i) regarding $\epsilon$

If $C = 0$ (changing of Phase) (like in Condenser, evaporator or boiler), then put that in '$\epsilon$' formula,

$$\epsilon_{parallel} = 1 - e^{-NTU}$$
for $\epsilon_{counter} = 1 - e^{-NTU}$ $\left\{ \text{as in LMTD both are same for Phase Change here.} \right\}$

If C = 1 (equal capacity rate)

$$\epsilon_{parallel} = \left[\frac{1 - e^{-2NTU}}{2}\right]$$

$$\epsilon_{counter} = \frac{0}{0} \quad \{\text{as in LMTD}\}$$

from L' Hospital rule.

$$\epsilon_{counter\,(C=1)} = \frac{NTU}{1+NTU}$$

4/12/2011

GATE-1998 The hot fluid at 200°c enters a H.E. at a mass flow rate of $10^4$ kg/hr, its specific heat is 2000 J/kg-K, it is to be cooled by another fluid entering at 25°c with a mass flow rate of 2500 kg/hr and specific heat 400 J/kg-K. If the overall heat transfer coefficient based on outside area of 20 m² is 250 W/m²K. find the exit temperature of the hot fluid when the fluids are in parallel flow.

Here only 2 temp. are given so we cannot use LMTD so we use NTU method.

Ans

$$\dot{m}_h C_{ph} = \frac{10^4}{3600} \times 2000 = 5555.5 \text{ W/K}$$

$$\dot{m}_c C_{pc} = \frac{2500 \times 400}{3600} = 277.7 \text{ W/K}$$

as $\dot{m}_c C_{pc} < \dot{m}_h C_p$

$$\Rightarrow C = \frac{\dot{m}_c C_{pc}}{\dot{m}_h C_{ph}} = 0.05$$

$$NTU = \frac{UA}{(\dot{m}_c C_{pc})} = \frac{250 \times 20}{277.7} = 18$$

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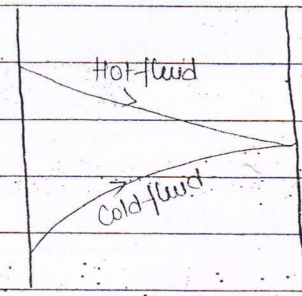
$$\epsilon_{parallel} = \frac{1-e^{-(1+C)NTU}}{1+C}$$

$$= \frac{1-e^{-(1+0.05)\times 18}}{1+0.05}$$

$$= 0.952$$

$\Rightarrow$ as $\dot{m}_c C_{pc} < \dot{m}_h C_{ph}$

$$\Rightarrow \epsilon = \frac{T_{ce} - T_{ci}}{T_{hi} - T_{ci}}$$




$$\Rightarrow 0.952 = \frac{T_{ce} - 25}{200 - 25}$$

$$\Rightarrow T_{ce} = 191.67^\circ C$$

from energy balance eq$^n$

$$\dot{m}_h C_{ph} \times (T_{hi} - T_{he}) = \dot{m}_c C_{pc} (T_{ce} - T_{ci})$$

$$\Rightarrow 5555.5 (200 - T_{he}) = 277.7 (191.67 - 25)$$

$$T_{he} = 191.67^\circ C$$

$T_{he} = T_{ce}$ (it is an uncommon problem)

I.E.S-2005 A process industry employs a counterflow heat exchanger to cool 0.8 kg/s of oil ($C_p = 2.5$ kJ/kg-K) from 120°C to 40°C by the use of water entering at 20°C. The overall heat transfer coefficient is estimated to be 1600 W/m²K. It is assumed that the exit temperature of water will not exceed 80°C. Using NTU method and taking NTU = 4 in this case Calculate.

(a). mass flow rate of water

(b). Surface area required

(c). Effectiveness of the heat exchanger.

Ans:

$T_{hi} = 120°c$

$T_{he} = 40°c$

$T_{ci} = 20°c$

$T_{ce} = 80°c$

$$\Rightarrow \dot{m}_h C_{ph} (120 - 40) = \dot{m}_c C_{pc} (80 - 20)$$

$$\Rightarrow \dot{m}_h C_{ph} (80) = \dot{m}_c C_{pc} (60)$$

$$\Rightarrow \frac{\dot{m}_c C_{pc}}{\dot{m}_h C_{ph}} = \frac{8}{6}$$

$$\Rightarrow \dot{m}_h C_{ph} < \dot{m}_c C_{pc}$$

$$\Rightarrow C = \frac{\dot{m}_h C_{ph}}{\dot{m}_c C_{pc}} = \frac{6}{8} = 0.75$$

$$\cancel{NTU} = \frac{UA}{\cancel{NTU}} \quad NTU = \frac{UA}{(\dot{m}C_p)_{smaller}} \Rightarrow NTU = \frac{UA}{\dot{m}_h C_{ph}}$$

$$4 = \frac{1600 \times A}{0.8 \times 2500} \Rightarrow \cancel{A = 10.5\,m^2} \quad A = 5\,m^2 \text{ Ans.}$$

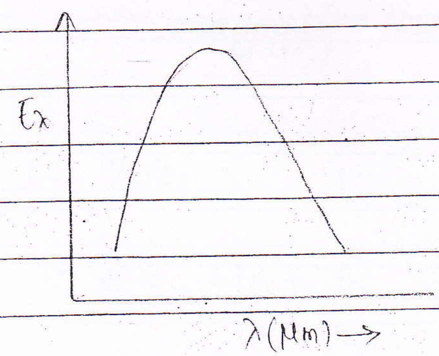
$$\Rightarrow 0.8 \times 2500 (80) = \dot{m}_c \times 4200 \times 60$$

$$\Rightarrow \dot{m}_c = 0.635 \text{ kg/s Ans.}$$

$$\epsilon = \frac{T_{hi} - T_{he}}{T_{hi} - T_{ci}} = \frac{120 - 40}{120 - 20}$$

# Radiation Heat Transfer

Any high temp. radiation is of Short wavelength (like solar radiation)
Low temp. radiation is of Long wavelength.




- All bodies at all temperatures emit thermal radiation, the rate of emission being a strong function of absolute temp. of body

- All high temp. bodies like Sun and flue gases burning in a furnace emit radiation at short wavelengths where as low temp. bodies emit at long wavelengths.

- Any Surface shall emit radiation over a certain range of wavelengths

Fundamental Definitions

- Total hemispherical Emissive Power ($E$) ($W/m^2$)

It is defined as the radiation energy emitted from a surface (by virtue of its temperature) in all possible hemispherical dirxn's per unit area per unit time integrated over all the wavelengths of emission. (hemisphere लिए sphere नहीं क्योंकि differential area नीचे की तरफ emit नहीं कर पाएगा but only in hemispherical dirxn).

- Total emissivity ($\epsilon$)

$\epsilon$ is defined as ratio b/w total hemispherical emissive power of a non black surface and total hemispherical emissive power of a black surface both being at the same temperature. $\epsilon = \frac{E}{E_b}$

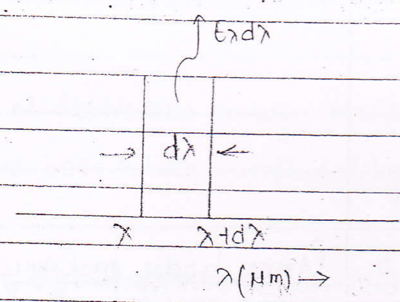
- **Black Body :-** A black body is a body which absorbs all the thermal radiation incident upon it. A black body is not only a perfect absorber but also is an ideal emitter.

(Black body)

A narrow aperture or Cavity in a hollow Container, all the energy absorbed after multiple reflections.

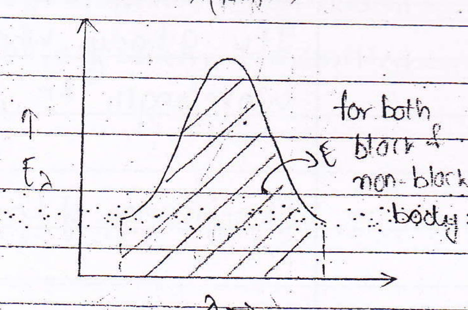
- **Monochromatic or Spectral emissive power**

$E_\lambda$ is defined as a quantity when multiplied by $d\lambda$ shall give the radiation energy emitted from a surface per unit time per Unit area in the wave length region $\lambda$ to $\lambda + d\lambda$, where $d\lambda$ is differential small increment in $\lambda$.




$$E_\lambda = f(\lambda) \rightarrow \frac{J}{\text{Sec}\, m^2 \mu m} = \frac{W}{m^2 \mu m}$$

$$E = \int_0^\infty E_\lambda \, d\lambda \;\; W/m^2$$




** for any given Surface and temperature, $E_\lambda$ changes with $\lambda$.

- **Monochromatic emissivity ($\epsilon_\lambda$)**

It is defined as the ratio b/w monochromatic emissive power of a non-black surface and monochromatic emissive power of a black surface both being at the same temperature and also at the same wavelength.

$$\epsilon_\lambda = \cancel{E_\lambda} \; \frac{E_\lambda}{E_{b\lambda}}$$

$$\epsilon = \frac{E}{E_b} = \frac{\int_0^\infty E_\lambda \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda}$$

and $\epsilon_\lambda = \dfrac{E_\lambda}{E_{b\lambda}} \Rightarrow E_\lambda = \epsilon_\lambda E_{b\lambda}$

$$\Rightarrow \epsilon = \frac{\int_0^\infty \epsilon_\lambda E_{b\lambda} \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda}$$

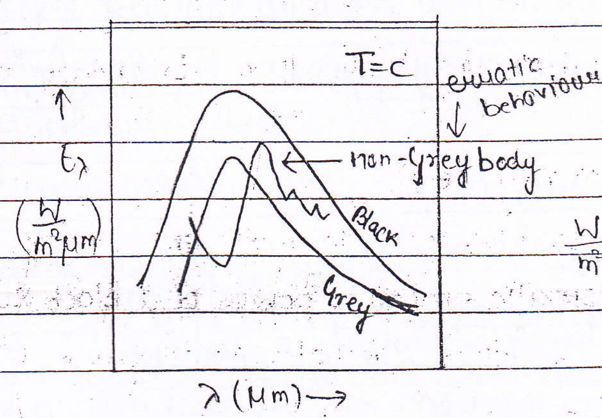
If $\epsilon_\lambda \neq f(\lambda)$ i.e. $\epsilon_\lambda$ = Constant

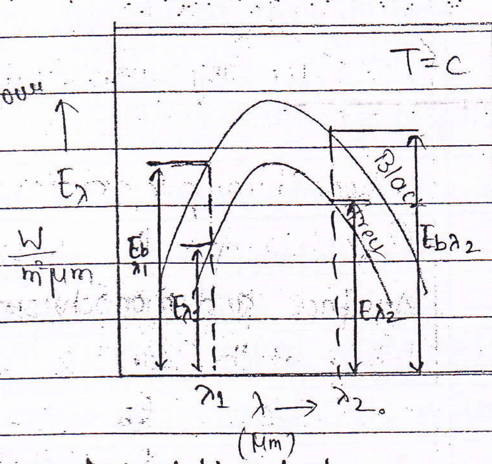
then $\epsilon = \epsilon_\lambda \dfrac{\int_0^\infty E_{b\lambda} \, d\lambda}{\int_0^\infty E_{b\lambda} \, d\lambda} \Rightarrow \epsilon = \epsilon_\lambda$

## Grey body or Grey Surface

It is a body whose monochromatic emissivity is independent of wavelength i.e. $\epsilon_\lambda \neq f(\lambda) \Rightarrow \epsilon_\lambda$ = Constant

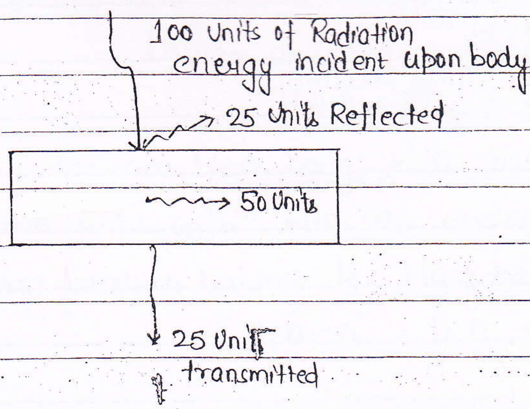
## Significance of Grey body




The grey body has same pattern as that of black body.

0 Absorptivity ($\alpha$), Reflectivity ($\rho$), Transmissivity ($\tau$)




Absorptivity ($\alpha$) = $\frac{50}{100} = 0.5$ fraction of Radiation energy incident upon a surface which is aborbed by it

Reflectivity ($\rho$) = $\frac{25}{100} = 0.25$ fraction of Radiation energy incident upon a surface which is reflected by it

Transmissivity ($\tau$) = $\frac{25}{100} = 0.25$ fraction of Radiation energy incident upon a surface which is transmitted through it

$\boxed{\alpha + \rho + \tau = 1}$ (for any surface)

$\boxed{\alpha = 1}$ (for black body)

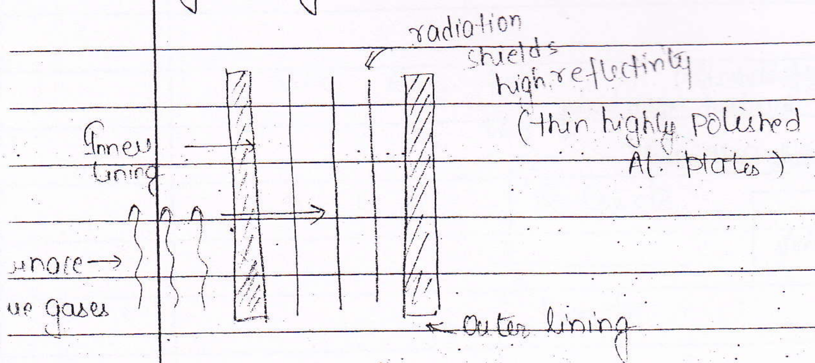
$\boxed{\tau = 0}$ (for opaque body)

$\boxed{\rho = 1}$ (for white body)

** The above properties mentioned ($\alpha, \rho, \tau$) change with wavelength of incident radiation and surface roughness and also with temperature.

Glasses are transparent to short wave length but is opaque to long wavelength.

Window glass of a Car is transparent to the short wavelength Solar Radiation but becomes opaque to the long wavelength re-radiation given by the Car.




## Laws of Thermal Radiation

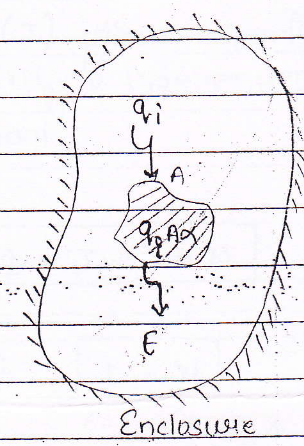
o **Kirchoff's law**

The law states that whenever a body is in thermal equilibrium with its surrounding, its emissivity is equal to its absorptivity.

$$\epsilon = \alpha$$

Proof:-

Consider a small sample of non-black body, having area A kept in an enclosure and allow the body to come into equilibrium with its enclosure.




Let $q_i$ be the incident radiation falling upon the body per unit area per unit time.

Total incident Radiation on body $= (q_i A)$ Watts

If $\alpha$ = absorptivity of body

then Radiation energy absorbed by body $= q_i A \alpha$

If $E$ is the emissive power of the body, then total energy emitted by body $= EA$

for steady state thermal eqlb$^m$ of sample energy balance for sample gives

Energy absorbed = Energy emitted

$$q_i A\alpha = EA \quad \text{---} \quad (1)$$

replace the non-black body with an identical black body and allow this to come into eqlb$^m$ with the enclosure.

Writing energy balance for black body,

$$q_i A \times \alpha_b = E_b A \quad \text{---} \quad (11)$$

(1) ÷ (11)

$\alpha_b = 1$

$$\Rightarrow \alpha = \frac{E}{E_b}$$ (both are in thermal eqlb$^m$ so same temp by zeroth law of thermodynamics)

$$\Rightarrow \alpha = \frac{E}{E_b} = \epsilon$$

proved

So, a good absorber is always a good emitter.

0 Planck's Law of Radiation

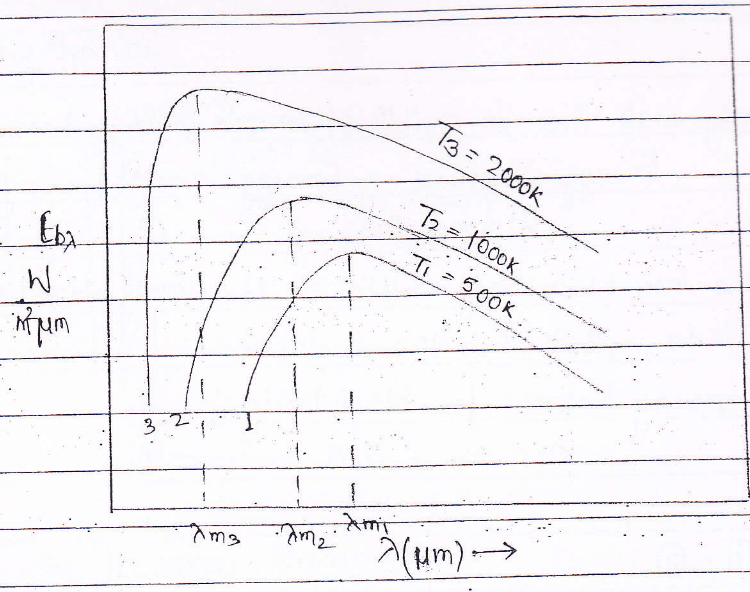
The law states that monochromatic or spectral emissive power of a black body depends upon the absolute temp. of the body and also on wavelength of emission. It is defined for black body.

$$\Rightarrow E_{b\lambda} = f(\lambda, T)$$

$$\Rightarrow E_{b\lambda} = \frac{2\pi C_1}{\lambda^5\left(e^{\frac{C_2}{\lambda T}} - 1\right)} \quad \text{Watts}/m^2\mu m$$

$C_1$ & $C_2$ are Constants.

The above functional relationship can be graphically plotted as




At a given absolute temp. of black body as wavelength $\lambda$ increases, monochromatic emissive power of body also increases reaches a maximum and then decreases. Let $\lambda_m$ be the wavelength at which $E_{b\lambda}$ is maximum for a given absolute temp. of black body.

As absolute temp. of black body increases $\lambda_m$ decreases i.e. with increase of temp. of the body most of the radiation is shifted to shorter wavelengths.

$$\lambda_m \propto \frac{1}{T}$$

$$\Rightarrow \lambda_m T = \text{Constant} \quad (\text{Wein's displacement law})$$

$$= 0.0029 mK \quad (2.89 \times 10^{-3} mK)$$

$$\therefore \text{from fig.} \quad \frac{\lambda_{m_3}}{\lambda_{m_1}} = \frac{500}{2000} = \frac{1}{4}$$

$$\frac{\text{Area Under } T_3}{\text{Area Under } T_1} = \frac{(E_b) \text{ at } T_3}{(E_b) \text{ at } T_1} = \left(\frac{T_3}{T_1}\right)^4 = 256$$

## Stefan-Boltzmann Law

The law state that total emissive power of a black body is directly proportional to the 4th power of absolute temp. of body.

$$E_b \propto T^4 \quad (T \text{ in } K)$$

$$\Rightarrow E_b = \sigma T^4 \ W/m^2$$

$$\sigma = 5.67 \times 10^{-8} \ W/m^2 K^4 \quad (\text{Stefan - Boltzman Constant})$$

Proof:-

$$E_b = \int_0^\infty E_{b\lambda} \, d\lambda$$

$$= \int_0^\infty \frac{2\pi C_1}{\lambda^5 (e^{C_2/\lambda T} - 1)} \, d\lambda$$

$$= \sigma T^4 \ W/m^2 K^4$$

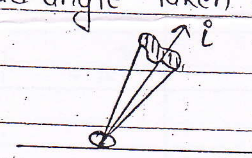
Thus both Wein's displacement law & Stefan Boltzmann law are obtained from plonk's law of radiation.

## Lambert's Cosine Law

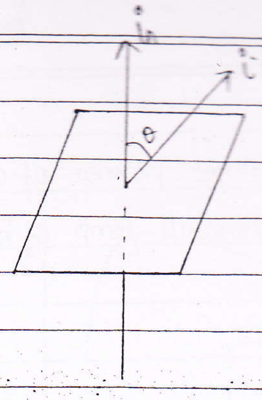
Intensity of Radiation (i) :- ~~It is defined as~~ Intensity of radiation along a given dirn is defined as the radiation energy emitted by the body per unit time per unit area and per unit solid angle taken about that dirn.

$$i = \frac{dE}{d\omega} \ W/m^2 \text{ steradian}$$

~~solid angle measured in steradian~~




$$\text{Total Emissive power } (E) = \int_0^{\pi} i \, d\omega$$

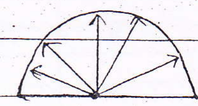



$i = i_n \cos\theta$

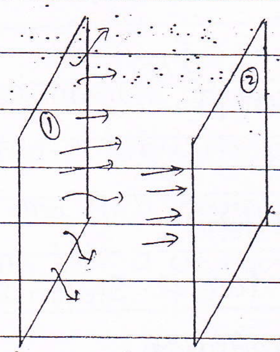
$i_n \rightarrow$ Normal intensity of radiation

$i \rightarrow$ Intensity of Radiation along a dirxn making angle $\theta$ with normal.

Diffusive surface is one whose intensity of radiation is the same in all directions (independent of direction)



Shape factor (OR) Configuration factor (OR) View factor



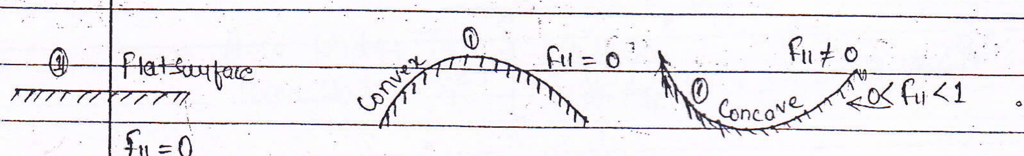

100 units of energy leaving ①

60 units of energy reaching ②

$F_{12}$ = Shape factor of ① w.r.t ②

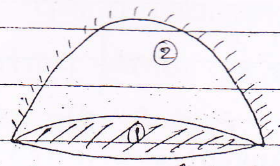
= Fraction of Radiation energy leaving surface ① which reaches surfaces ②. $= \frac{60}{100} = 0.6$

$F_{21}$ = fraction of radiation energy leaving surface ② and reaches surface ①.



When one body or surface is Completely surrounded by another body the shape factor of the inner body w.r.t Outer is equal to 1.

Eg:-

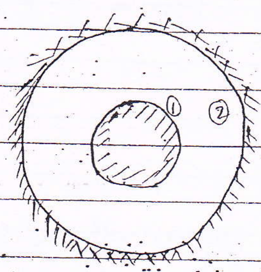


Circular Plate

$F_{12} = 1$

$0 < F_{21} < 1$ , $0 < F_{22} < 1$

$F_{11} = 0$



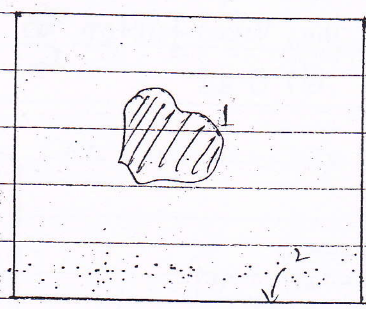
$F_{12} = 1$

$0 < F_{21} < 1$

$F_{11} = 0$ , $0 < F_{22} < 1$

Two Concentric spherical Surfaces

When a large body is kept in a large enclosure or environment.

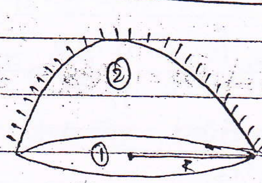


$F_{12} = 1$

$0 < F_{21} < 1$

Reciprocity Relation b/w shape factors

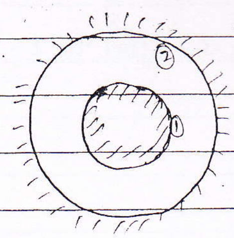
$$\boxed{A_1 F_{12} = A_2 F_{21}}$$



$$F_{21} = \frac{A_1}{A_2} = \frac{\pi R^2}{2\pi R^2} = \frac{1}{2}$$

$$\Rightarrow F_{22} = 1 - \frac{1}{2} = 0.5$$

** This relation holds good between any two surfaces even when there are more than 2 no. of surfaces involved in radiation heat exchange.




$$F_{21} = \left(\frac{R_1}{R_2}\right)^2$$

$$\Rightarrow F_{22} = 1 - \left(\frac{R_1}{R_2}\right)^2$$

If there are total 'n' no. of surfaces involved in a radiation heat exchange then

$$F_{11} + F_{12} + F_{13} + \dots + F_{1n} = 1$$

$$F_{21} + F_{22} + F_{23} + \dots + F_{2n} = 1$$

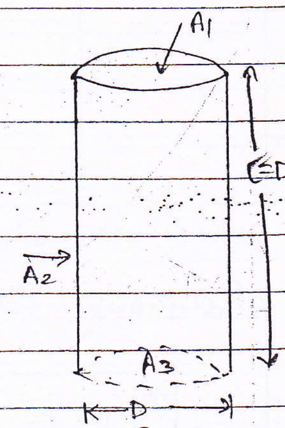
$$\vdots$$

$$F_{n1} + F_{n2} + \dots + F_{nn} = 1$$

If surface is flat or convex, self shape factor is zero.

A.T.E-2001 For the circular tube of equal length and diameter shown below, the view factor $F_{13} = 0.17$, the view factor $\cancel{F_{22}}$ $F_{21}$ in this case will be

(a) 0.17 (b) 0.21 (c) 0.79 (d) 0.83




Ans.

$$F_{11} + F_{12} + F_{13} = 1$$

$$F_{12} + F_{13} = 1 \Rightarrow \cancel{F_{12} = 0.83}$$

$$F_{12} = 1 - F_{13} = 0.83$$

$$F_{22} + F_{23} + F_{21} = 1$$

$$F_{32} + F_{31} = 1$$

$$A_1 F_{12} = A_2 F_{21}$$

$$\cancel{A_3 F_{32} = A_2 F_{23}}$$

$$\Rightarrow \frac{\pi}{4} D^2 \times F_{12} = \pi \times D \times L \times F_{21}$$

$$\cancel{\Rightarrow \frac{\pi}{4} D^2 F_{32} = 2\pi D F_{23}}$$

$$\Rightarrow F_{21} = \frac{0.83}{4} = 0.21 \text{ Ans.}$$

$$\cancel{\Rightarrow F_{32} = 8F_{23}}$$

$$\cancel{A_1 F_{13} = A_3 F_{31}}$$

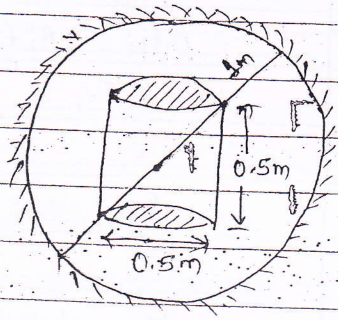
$$\cancel{A_3 F_{31}} = A_2 F_{13}$$

$$\Rightarrow \qquad \Rightarrow F_{31} = F_{13}$$

G.A.T.E-2005 A Solid Cylinder (Surface 2) is located at the centre of a hollow sphere (Surface 1). The diameter of the sphere is 1m while the cylinder has a diameter and length of 0.5m each. The radiation Configuration factor $F_{11}$ is

(a). 0.375 (b) 0.625 (c) 0.75 (d) 1

Ans: $F_{11} + F_{12} = 1$ —— (1)




and $F_{21} = 1$

By reciprocity theorem

$\Rightarrow A_1 F_{12} = A_2 F_{21}$

$$\Rightarrow \pi \times 1^2 \times F_{12} = \left(\pi \times 0.5 \times 0.5 + 2 \times \pi \left(\frac{0.5}{2}\right)^2\right) \times F_{21}$$

$$\Rightarrow F_{12} = 0.375\, F_{21}$$

as $F_{21} = 1 \Rightarrow F_{12} = 0.375$

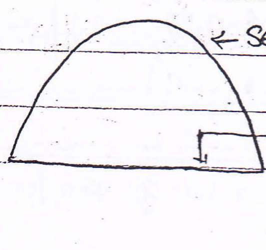
$\Rightarrow F_{11} + F_{12} = 1$

$\Rightarrow F_{11} + 0.375 = 1$

$\Rightarrow F_{11} = 0.625$

5/12/2011

I.ES-2011 For a very long semicircular duct having surface area of $A_2$ and $A_1$ as shown below



$$\boxed{I_n = \frac{E_b}{\pi}}$$

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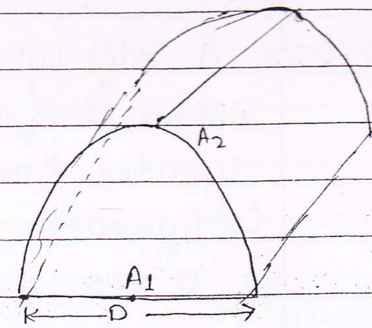
Determine shape factors $f_{12}$, $f_{21}$ & $f_{22}$




Ans. $f_{11} + f_{12} = 1$

$f_{11} = 0$ (flat surface)

$\Rightarrow f_{12} = 1$

$f_{21} + f_{22} = 1$

~~verton~~ Very long so that some energy don't escape from the semicircular x-s/c.

$A_1 f_{12} = A_2 f_{21}$

$\Rightarrow \quad f_{21} = \frac{A_1 f_{12}}{A_2} = \frac{A_1}{A_2}$

$f_{22} = 1 - \frac{A_1}{A_2}$

for Unit length

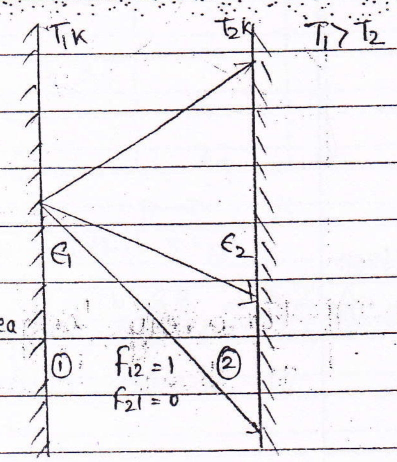
$\Rightarrow \quad f_{22} = 1 - \frac{D \times L}{\frac{\pi D}{2} \times L} = 1 - \frac{2}{\pi}$

$\Rightarrow \quad f_{21} = \frac{2}{\pi}$

Case I $\rightarrow$ Radiation Heat Exchange b/w two very large parallel plane surfaces

Assume

- Surfaces are grey & diffuse
- Isothermal surfaces
- Steady state conditions




Net radiation heat exchange b/w 1 & 2 / unit area

$$\Rightarrow \quad \frac{q_{1-2\,net}}{A} = \frac{\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)} \quad W/m^2$$

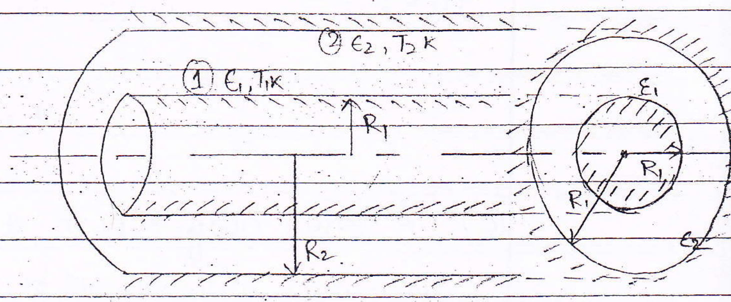
For very large area we go ~~to~~ for unit area calculating flux.

Vaccum Prevailing

Case 2 → Radiation heat Exchange b/w two very long Concentric Cylindrical Surfaces

Assume

- Grey and diffuse Surfaces
- Steady state Conditions
- Vaccum prevailing




$\therefore F_{12} = 1$

$0 < F_{21} < 1$

Net radiation heat exchange b/w ① and ②

$$= q_{1-2 \, net} = \frac{\sigma (T_1^4 - T_2^4) A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)} \text{ Watts}$$

for a length of Lm,

$A_1 = 2\pi R_1 L$

$A_2 = 2\pi R_2 L$

$$\Rightarrow \frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)$$

** The above formulae given are applicable to net radiation heat exchange b/w any two surfaces whenever one shape factor is equal to 1 and the other shaper is less than 1.

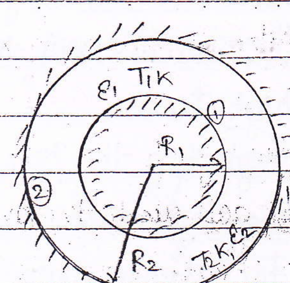
E.g (a). Two Concentric Spheres

$F_{12} = 1$

$F_{12} < 1$

$$\frac{A_1}{A_2} = \left(\frac{R_1}{R_2}\right)^2$$

$A_1 = 4\pi R_1^2$

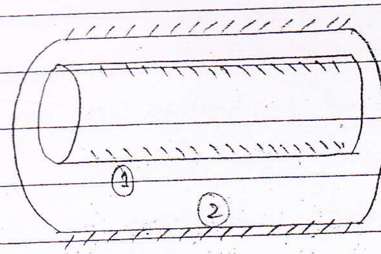



Applications in Cryogenic, When fluids are kept b/w 0K and 133K where all gases are in liquid form are kept for a long time. Sphere

(b) Eccentric long Cylindrical Surfaces

$F_{12} = 1$

$F_{21} < 1$



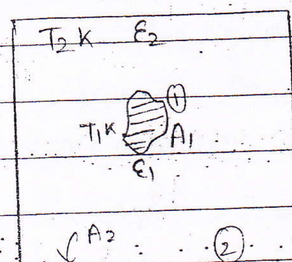

(c) A small body kept in a large enclosure

$F_{12} = 1$

$F_{21} < 1$

When $A_1 <<< A_2$

$\Rightarrow \frac{A_1}{A_2} \rightarrow 0$




Then,

$(q_{1-2})_{net}$ = Net radiation heat exchange

$$= \frac{\sigma (T_1^4 - T_2^4) A_1}{\frac{1}{\varepsilon_1} + \cancel{\frac{A_1}{A_2}}^{\,0} \left(\frac{1}{\varepsilon_2} - 1\right)} = \sigma (T_1^4 - T_2^4) A_1 \varepsilon_1$$

In General, take inner Surface as 1 and Outer 2

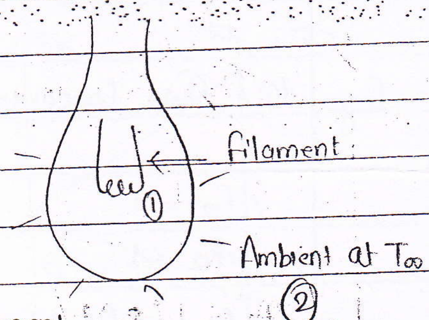
Practical e.g.

Filament area very small

Tungsten filament to withstand high temp.

neglecting the role of glass,

Inert gas used to avoid oxidation of filament




$F_{12} = 1$ $\quad \frac{A_1}{A_2} \rightarrow 0$

$F_{21} < 1$

for steady state conditions of filament,

Writing energy balance,

$$\text{Power of bulb} = q_{net\ filament\_amb} = \sigma \left[ T_{fil}^4 - T_{amb}^4 \right] \varepsilon_{fil} \times A_{filament}$$

I.E.S 1998 The filament of a 75 W light bulb may be considered a black body radiating in to a black enclosure at 70°C. The filament diameter is 0.10 mm and the length is 5cm, Considering only radiation, determine the filament temperature.

Ans. bulb as black body $\Rightarrow \epsilon = 1$

- for Steady state
- Also the area Compared to surrounding very smal.

$\Rightarrow$ $\cancel{\frac{\pi}{4}} \times \cancel{(0.1\times10^{-3})^2}$

$$\pi \times (0.1\times10^{-3}) \times 5\times10^{-2} \times 5.67\times10^{-8} \times (T^4 - \cancel{70^4}\,T_\infty) = 75$$

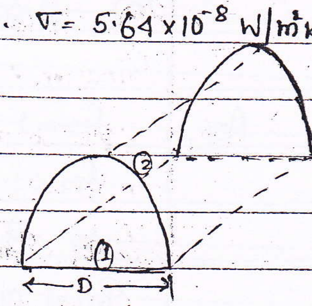
$$\Rightarrow 1.57 \times 10^{-13} \times 5.67 \times (T^4 - 343^4) = 75$$

$$\Rightarrow T_{filament} = 3029.8\ K$$

~~GATE 2002~~
I.E.S 2002 The cross-section of a very long black body enclosure consists of a semicircle with its diameter 'D' as base. The temperature of semi-circle is 1000K and that of diameter is 500K. Determine the shape factors for diameter - semicircle combination and the radiation heat transfer rate per unit width (in terms of D). $\sigma = 5.64\times10^{-8}\ W/m^2K^4$.

Ans. $F_{11} = 0$

$\Rightarrow F_{12} = 1$




$A_1 F_{12} = A_2 F_{21}$

$$\Rightarrow F_{21} = \frac{A_1}{A_2} = \frac{D\times L}{\pi\times\frac{D}{2}\times L} = \frac{2}{\pi}$$

$$\Rightarrow F_{22} = 1 - \frac{2}{\pi}$$

Here $f_{12} = 1$

and $f_{21} < 1$

$\therefore$ $q_{net}$ formula can be used

$\Rightarrow$ $(q_{1-2})_{net}$ per unit width

$$= \frac{\sigma(T_1^4 - T_2^4)\,A_1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\epsilon_2} - 1\right)}$$

$\epsilon_1 = 1$ & $\epsilon_2 = 1$ (as both are black bodies)

$= \sigma(T_1^4 - T_2^4) \times W \times D$ Watts

Put $W = 1m$.

$\Rightarrow$ $5.67 \times 10^{-8}(500^4 - 1000^4) \times 1 \times D$ Watts/m

$= -52875\,D$ W/m

-ve sign shows net heat exchange from ② to ①

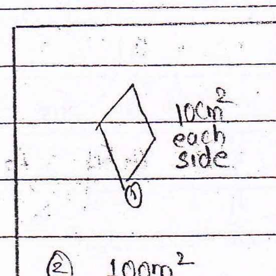
A.T.E - 2003 A plate having $10\,cm^2$ area each side is hanging in the middle of a room of $100m^2$ total surface area, the plate temp. and emissivity are respectively 800K and 0.6, the temperature and emissivity values for the surfaces of the room are 300K and 0.3 respectively. $\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$. The total heat loss from the two surfaces of the plate is

(a) 13.66 Watt (b) 27.32 W (c) 27.87 W (d) 13.68 W

Ans. $f_{12} = 1$

$f_{21} < 1$

So, $q_{net}$ formula is applicable




Total area of plate $= (2 \times 10)$

$= 20cm^2 = A_1$

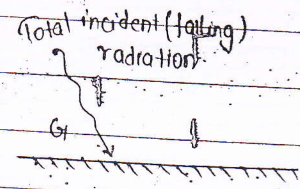
$A_2 = 100\ m^2$

$(q_{1-2})_{net}$ = Total radiation heat loss from plate

$$= \frac{5.67\times10^{-8}(800^4-300^4)\times 20\times10^{-4}}{\frac{1}{0.6}+\frac{20\times10^{-4}}{1000}\left(\frac{1}{0.3}-1\right)} = 27.32 \text{ Watts}$$

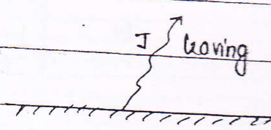
## Radiation Networks

Incident
Irradiation & Radiosity
$G$ $\quad$ $J$




Irradiation is defined as the total radiation energy incident upon a surface per unit time and per unit area, having units $W/m^2$.

G – Girta है जो
i.e. Irradiation




Radiosity (J) is defined as the total radiation energy leaving a surface per unit time and per unit area.

E, G & J have same units $(W/m^2)$

$$J = \text{Emitted Energy} + \text{Reflected part of incident energy}$$
$$= E + \rho G$$

for any surface

$$\alpha+\rho+\tau = 1$$

for opaque surface, $\tau = 0$

$$\Rightarrow \alpha+\rho = 1$$
$$\Rightarrow \rho = 1-\alpha$$
$$\Rightarrow \rho = 1-\epsilon \quad \text{(by Kirchhoff's law)}$$

$$\Rightarrow J = E + (1-\epsilon)G$$
$$= \epsilon E_b + (1-\epsilon)G$$

If A is the surface area of the body, the net radiation heat exchang b/w the body and all of its surrounding is given by

$$(q_{net})_{\text{Surface - Surroundings}} = (JA) - (GA)$$

$$= (J - G)A \text{ Watts}$$

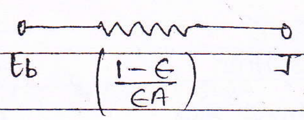
$$= A\left[\epsilon E_b + (1-\epsilon)G - G\right] \text{ Watts}$$

Eliminating G, we get

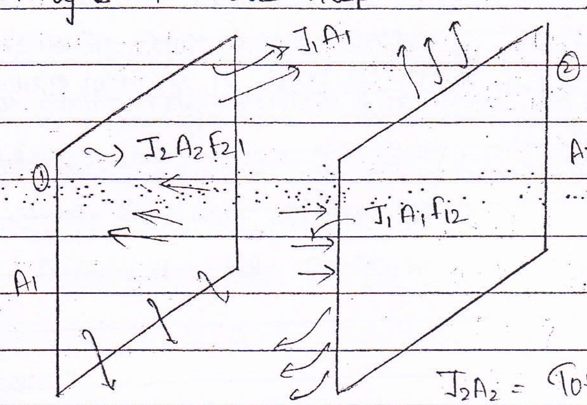
$$(q_{net})_{\text{surface surr}} = \frac{E_b - J}{\left(\frac{1-\epsilon}{\epsilon A}\right)} \leftarrow \text{Surface resistance}$$

Where $G = \left(\frac{J - \epsilon E_b}{1-\epsilon}\right)$

Drawing equivalent circuit for heat current,




Consider two finite grey surfaces of areas $A_1$ and $A_2$ and emissivity is $\epsilon_1$ and $\epsilon_2$ resp.

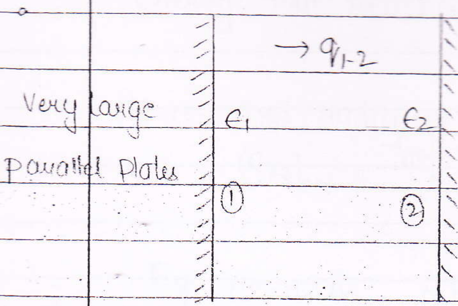


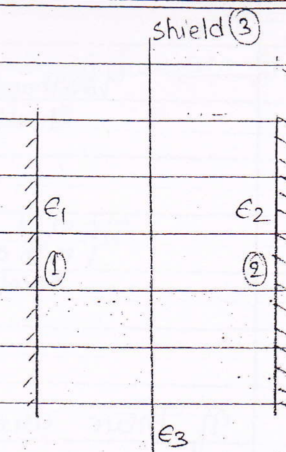

$J_2A_2$ = Total energy leaving ②

$J_1A_1$ = Total energy leaving ①

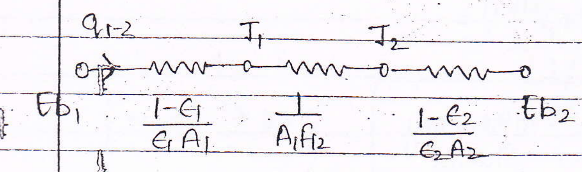
Out of total radiation energy leaving surface ① the energy that reaches surface ② is $J_1A_1F_{12}$

## Radiation Shields

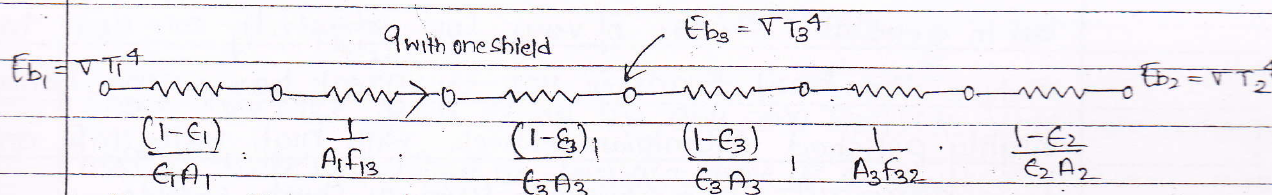



Shield introducing 2 surface resistance from both sides.




Thermal circuit diagram with shield.




Put $A_1 = A_2 = A_3 = 1m^2$, $F_{13} = F_{32} = 1$ (very large)

$$(q_{\text{with one shield}}) = \frac{\sigma (T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_1} + \frac{2}{\epsilon_3} + \frac{1}{\epsilon_2} - 2\right)} \quad W/m^2$$

Each radiation shield shall bring in 3 additional resistances into the network out of which 2 are surface resistances and one is space resistance.

** If 'n' shields are kept b/w the plane surface the total no. of surface resistances in the radiation network is $(2n+2)$ and the total no. of space resistances in the network is $(n+1)$.

If all emissivities are equal i.e.

$$\epsilon_1 = \epsilon_2 = \epsilon_3 = \epsilon$$

Then,

$$q_{\substack{\text{without}\\ \text{Shield}}} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{2}{\epsilon} - 1} \quad W/m^2$$

$$q_{\substack{\text{with one}\\ \text{shield}}} = \frac{\sigma(T_1^4 - T_2^4)}{4/\epsilon - 2} \quad W/m^2$$

If there are ~~not~~ total 'n' no. of Shields kept all having the same emissivity as those of planes, then

$$\boxed{q_{\substack{\text{with } n\\ \text{Shields}}} = \frac{1}{(n+1)} \times q_{\substack{\text{without any}\\ \text{Shields}}}} \quad \{\text{all } \epsilon' \text{ same}\}$$

$\downarrow \epsilon_3 = 1 - e_3 \uparrow$ by polishing

but in general, shields of very low emissivity are used to reduce the heat transfer rate by great percentage. hence highly polished aluminium sheets with high reflectivity and low emissivity are generally used as ~~sheets~~ shields

E.S-2005 An industrial furnace employs a hollow brick lining, the inside and outside surfaces of hollow brick lining are maintained at 900k and 430k by placing the radiation shields in b/w the hollow space, the heat lost to the furnace surroundings at 300k is both by radiation and natural convection. By sketching the arrangement calculate the no. of radiation shields needed, the emissivity of wall and the shields may be taken as 0.85. The convective heat transfer coefficient is governed by expression $h = 1.5(\Delta T)^{0.33}$

(20 marks)

Similarly, ~~ts~~ Out of total radiation energy leaving surface ②, the energy that reaches surface ① is $J_2 A_2 F_{21}$

Hence, net radiation heat exchange between ① and ②

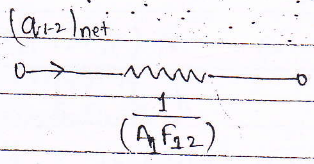
$$(q_{1-2})_{net} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

By reprocity, $A_1 F_{12} = A_2 F_{21}$

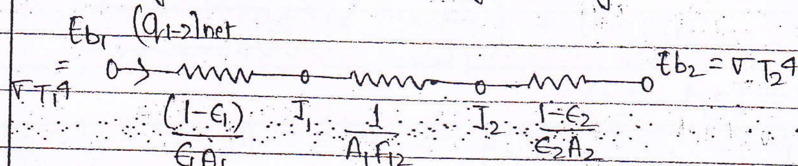
$$\Rightarrow (q_{net})_{1-2} = (J_1 - J_2) A_1 F_{12} \text{ watts}$$

$$= \frac{(J_1 - J_2)}{(1/A_1 F_{12})} \leftarrow \text{space resistance}$$

∴ the equivalent circuit is :-




Space resistance shall exist b/w any two surfaces which are exchanging heat by radiation, whereas for every surface there is a surface resistance existing. Hence, the complete radiation network for radiation heat exchange b/w any two surfaces is given by



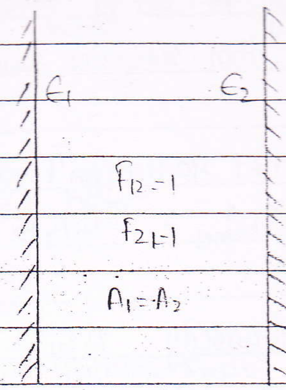

$$\Rightarrow (q_{1-2})_{net} = \frac{E_{b_1} - E_{b_2}}{\Sigma R_{th}} = \frac{\sigma (T_1^4 - T_2^4)}{\frac{1-\epsilon_1}{\epsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\epsilon_2}{\epsilon_2 A_2}}$$

In case surfaces are black, then $\epsilon_1 = \epsilon_2 = 1$

$$(q_{1-2})_{net} = \frac{\sigma (T_1^4 - T_2^4)}{1/A_1 F_{12}}$$

Eg (a) For large parallel planes

$$q_{1-2} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$




$$\Rightarrow q_{1-2} = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1 A_1} - \cancel{\frac{1}{A_1}} + \cancel{\frac{1}{A_1}} + \frac{1}{\varepsilon_2 A_2} - \frac{1}{A_2}}$$

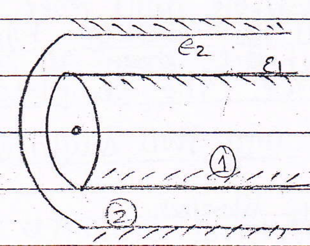
$$= \frac{\sigma(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1 A_1} + \frac{1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)}$$

$$= \frac{\sigma(T_1^4 - T_2^4) \times A_1}{\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)} \quad , A_1 = A_2$$

$$\Rightarrow \frac{\sigma_1(T_1^4 - T_2^4) \times A_1}{\left(\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1\right)}$$

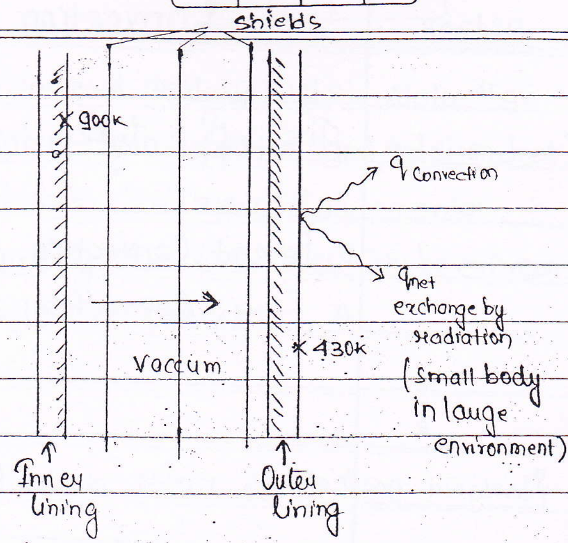
(b). For very long concentric cylindrical surfaces

$$(q_{1-2})_{net} = \frac{\sigma(T_1^4 - T_2^4)}{\left[\frac{1}{\varepsilon_1 A_1} - \cancel{\frac{1}{A_1}} + \cancel{\frac{1}{A_1 \times 1}} + \frac{1}{\varepsilon_2 A_2} - \frac{1}{A_2}\right]}$$




$$= \frac{\sigma(T_1^4 - T_2^4)}{\left[\frac{1}{\varepsilon_1 A_1} + \frac{1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right]}$$

$$= \frac{\sigma(T_1^4 - T_2^4) A_1}{\left[\frac{1}{\varepsilon_1} + \frac{A_1}{A_2}\left(\frac{1}{\varepsilon_2} - 1\right)\right]}$$




Ans.

$$h_{free} = 1.5\,(\Delta T)^{0.33}$$

$$= 1.5 \times (430-300)^{0.33}$$

$$= 7.476 \ W/m^2K$$

Put $A = 1m^2$ (as it is a large body)

$$q_{conv} = hA(\Delta T)$$

$$= 7.476 \times 1 \times (430-300)$$

$$= 971.93 \ W/m^2$$

$(q_{net})_{radiation}$ b/w outer lining & ambient air. $= \sigma(430^4 - 300^4) \times 0.85 \times 1$

$$= 1257.3 \ W/m^2$$

$(q_{net})_{radiation}$ b/w inner and outer lining $= q_{conv} + q_{\text{Radiation exchange from outer lining}} = 2229.2 \ W/m^2$

$\Rightarrow$ $(2n+2)$ Surface resistance and $(n+1)$ space resistance.

$$\Rightarrow \frac{\sigma\,(T_{inner}^4 - T_{outer}^4)}{(2n+2)\left(\frac{1-0.85}{0.85\times1}\right) + \frac{(n+1)\times1}{1\times1}} = 2229.2 \ W/m^2$$

$\Rightarrow$ $n = 11$ Shields Ans.

1/12/2011

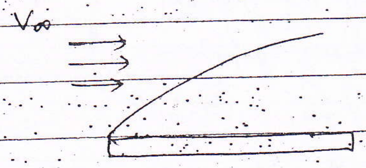
# Convection Heat Transfer

It is of 2 types

- Forced Convection
- Free Convection

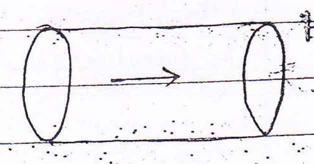
Forced Convection ( velocity of fluid is provided by external agenty .

Flow over flat plates

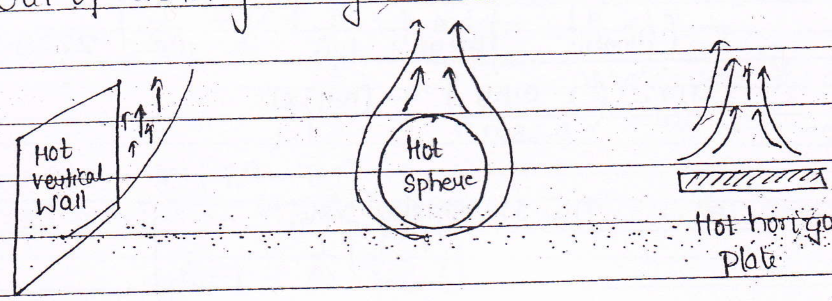



(External flow)

Flow through pipes or ducts



Free Convection:- Flow occurs naturally due to buoyancy forces arising out of density changes.




Hot horizontal plate

In any convection heat transfer the thermal energy gets transported by the moving fluid (macroscopic bulk motion)

In any forced convection heat transfer,

$$h = f(\vec{V}, D, \underbrace{\rho, \mu, C_p, K}_{\text{Thermophysical properties of fluid.}})$$ ( 7 no. of variables, 6 independent & 1 depend

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from Buckhingham-$\pi$ theorem of dimensional analysis, which states that if there are total 'n' no. of variables (both dependent and independent) and if all the variables put together contain 'm' no. of fundamental dimensions, then the relationship among the variables can be expressed in terms of $(n-m)$ no. of dimensionless $\pi$ terms.

Here $n = 7$

$4\ m = 4$ $(M, L, T, \theta(\text{temp}))$ {4 only in H.m.T due to temp. involved}

$\therefore$ no of $\pi$-terms $= n-m = 7-4 = 3$

let the $\pi$-terms be $\pi_1, \pi_2, \pi_3$

To get $\pi_1, \pi_2, \pi_3$

Choose 'm' no. of repeating variables

the repeating variables must be chosen in such a way that

(a). All of them put together contain all the fundamental dimensions.

(b). They themselves should not form a dimensionless group b/w them.

(c). They may be present in $\pi$ terms.

$\therefore$ Selecting $h, v, D, \rho$ as repeating variables

$h \rightarrow \dfrac{J}{\sec m^2 k} = \dfrac{N\cdot m}{\sec m^2 k} = \dfrac{MLT^{-2}}{TL\theta} = MT^{-3}\theta^{-1}$

$C_p \rightarrow \dfrac{J}{kg\,k} = \dfrac{Nm}{kg\,k} = \dfrac{ML^2T^{-2}}{M\theta} = L^2T^{-2}\theta^{-1}$

$\mu \rightarrow \dfrac{kg}{m.\sec} = (ML^{-1}T^{-1})$

$k \rightarrow \dfrac{W}{mk} = MLT^{-3}\theta^{-1}$

$V \rightarrow LT^{-1}$, $\rho \rightarrow ML^{-3}$

Then,

$$\pi_1 = \left[h^{a_1} V^{b_1} D^{c_1} e^{d_1}\right] \times \mu$$

$$\pi_2 = \left[h^{a_2} V^{b_2} D^{c_2} e^{d_2}\right] \times C_p$$

$$\pi_3 = \left[h^{a_3} V^{b_3} D^{c_3} e^{d_3}\right] \times K$$

To get $\pi_1$:-

$$M^0 L^0 T^0 \theta^0 = \left[M T^{-3} \theta^{-1}\right]^{a_1} \left[L T^{-1}\right]^{b_1} \left[M L^{-3}\right]^{d_1} \times M L^{-1} T^{-1} \times [L]^{c_1}$$

for mass (M) :- $0 = a_1 + d_1 + 1$

for length (L) - $0 = b_1 + c_1 - 3d_1 - 1$

for time (T) - $0 = -3a_1 - b_1 - 1$

for temp ($\theta$) - $0 = -a_1$

$\Rightarrow \quad d_1 = -1, \quad b_1 = -1, \quad c_1 = -1$

$$\Rightarrow \quad \pi_1 = \frac{\mu}{VDe}$$

To get $\pi_2$:-

$$M^0 L^0 T^0 \theta^0 = \left[M T^{-3} \theta^{-1}\right]^{a_2} \left[L T^{-1}\right]^{b_2} [L]^{c_2} \left[M L^{-3}\right]^{d_2} \times L^2 T^{-2} \theta^{-1}$$

$0 = a_2 + d_2$

$0 = b_2 + c_2 - 3d_2 + 2$

$0 = -3a_2 - b_2 - 2$ { here we always get exponents $-1$, 1 or 0 }

$0 = -a_2 - 1$

$a_2 = -1, \quad b_2 = 1, \quad c_2 = 0, \quad d_2 = 1$

$$\Rightarrow \quad \pi_2 = \frac{eVC_p}{K}$$

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$$M^0L^0T^0\theta^0 = [MT^{-3}\theta^{-1}]^{a_3}[LT^{-1}]^{b_3}[L]^{c_3}[ML^{-3}]^{d_3} \times MLT^{-3}\theta^{-1}$$

$\Rightarrow$ $a_3 + d_3 + 1 = 0$ (for mass)

$b_3 + c_3 - 3d_3 + 1 = 0$ (for length)

$-3a_3 - b_3 - 3 = 0$ (for time)

$-a_3 - 1 = 0$ (for temp.)

$\Rightarrow$ $a_3 = -1$

$d_3 = 0$

$c_3 = 0$

$b_3 = -1$

$$\Rightarrow \pi_3 = \frac{K}{hD}$$

Let $\pi_4 = \frac{1}{\pi_1} = \frac{VDe}{\mu}$ (we have taken $\frac{1}{\pi_1}$ as $\pi_4$ becoz $\frac{VDe}{K}$ is known and called as reynolds no.)

$$\pi_5 = \frac{1}{\pi_3} = \frac{hD}{K}$$

$$\pi_6 = \frac{\pi_1\pi_2}{\pi_3} = \frac{\mu C_p}{K}$$

$\therefore$ The functional relationship can be expressed as :-

$$\pi_5 = f(\pi_4, \pi_6)$$

$$\Rightarrow \frac{hD}{K} = f\left(\frac{VDe}{\mu}, \frac{\mu C_p}{K}\right)$$ in any forced convection heat transfer.

$\Rightarrow$ $Nu = f(Re, Pr)$ [Any forced Convection]

'Nu' is kept on left as we want to compute 'h'.

<u>Physical Significance of dimensionless number in forced Convection heat transfer</u>

0 Reynold's No(Re):- It is defined as the ratio b/w inertia forces and the viscous forces to which ~~the are~~ a flowing fluid is subjected.

i.e. $Re = \dfrac{\text{Inertia forces}}{\text{Viscous forces}} = \dfrac{V D \rho}{\mu} = \dfrac{VD}{\nu}$ ← D: Charac. dimension; $\nu$ ← K.V ($m^2/sec$)

Newton's law of ~~Cooling~~ Viscosity

$$\tau \text{ (viscous or shear stress)} = \mu \left(\frac{\partial u}{\partial y}\right) \text{ pascals}$$

↑ velocity gradient

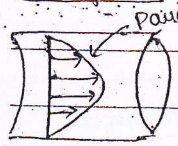
Viscosity is a mechanism of momentum transfer or momentum diffusion.

* * Viscosity is a mechanism of momentum transfer b/w fluid layers i.e. kinematic viscosity, a property of fluid indicates the rate at which momentum may diffuse through the fluid layers.

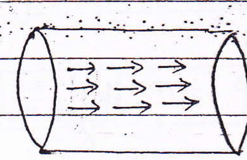
Reynold's no. use the criteria to tell whether a fluid flow is laminar or turbulent.

velocity dist.

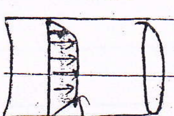
for flow through pipes (i.e. internal flows)



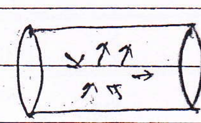

If $Re < 2000 \Rightarrow$ flow is laminar



flow occurs in laminae.




If $Re > 4000 \Rightarrow$ Flow is turbulent



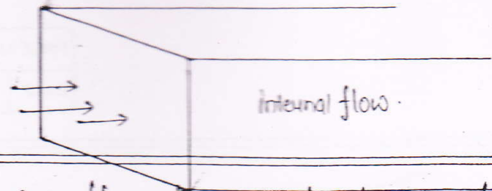
Continuous intermingling of fluid particles

$\nu_{air} \rightarrow 1.5 \times 10^{-5}\ m^2/sec.$

When in ~~nu~~ denominator (Re) easily crosses ~~R~~ 4000, so air flow is always turbulent.

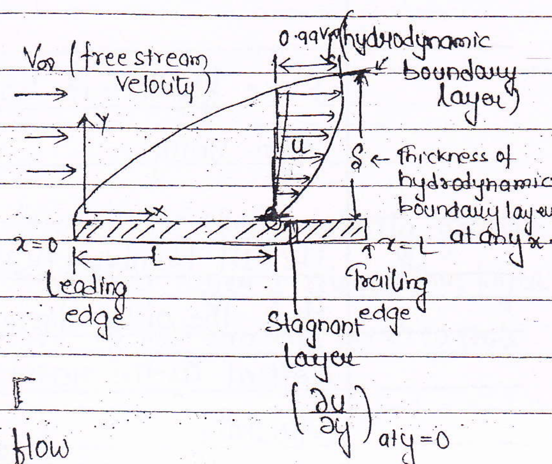
Very commonly the internal fluid flows are turbulent particularly in case of gases. Air flow in duct by default is turbulent.

$D = \frac{4 A_{x\text{-sect}}}{\text{Perimeter}}$




for turbulent flows there will be better heat transfer

## for flow over Flat plates (External flow)




Let $x$ = distance of any location from leading edge.

Local Reynold's no $= Re_x = \frac{V_\infty x \rho}{\mu}$

If $Re_x < 5\times10^5 \Rightarrow$ flow is laminar

If $Re_x > (6.5 \text{ to } 7\times10^5) \Rightarrow$ Turbulent flow

** In flow over flat plates commonly the flows are laminar.

$\delta = f(x)$ for laminar boundary layer, $\boxed{\delta = \frac{4.64x}{\sqrt{Re_x}}}$

At any given $x$,

At $y = 0 \Rightarrow u = 0$

At $y = \delta \Rightarrow u = 0.99V_\infty$

Hydrodynamic boundary layer is a thin region inside which there are velocity gradients present in the normal dirx^n to the plate.

$\tau_{wall}$ (local wall shear stress at any $x$) $= \mu\left(\frac{\partial u}{\partial y}\right)_{at\, y=0}$

$\boxed{\tau_w \propto \frac{1}{\sqrt{x}}}$

## Nusselt Number (Nu):-

$$Nu = \frac{hD}{K_{fluid}}$$

It resembles Biot No $= \frac{hs}{K_{solid}} < (0.1)$ for lumped heat analysis

$$Nu = \frac{D/kA}{1/hA} = \frac{\text{Conduction resistance offered by fluid if it were stationary}}{\text{Convection Resistance}}$$

Since fluids are bad conductors of heat, Nu is always greater than unity.

Prandtl Number (Pr):-

It is the only dimensionless number which is a property of fluid defined as the ratio b/w kinematic viscosity of fluid and its thermal diffusivity.

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c_p} = \frac{\mu c_p}{k} \quad \leftarrow J/kg\text{-}K$$

$\nu$ signifies the rate at which momentum diffusion occurs through fluid

$\alpha$ signifies the rate at which heat diffusion occurs through fluid.

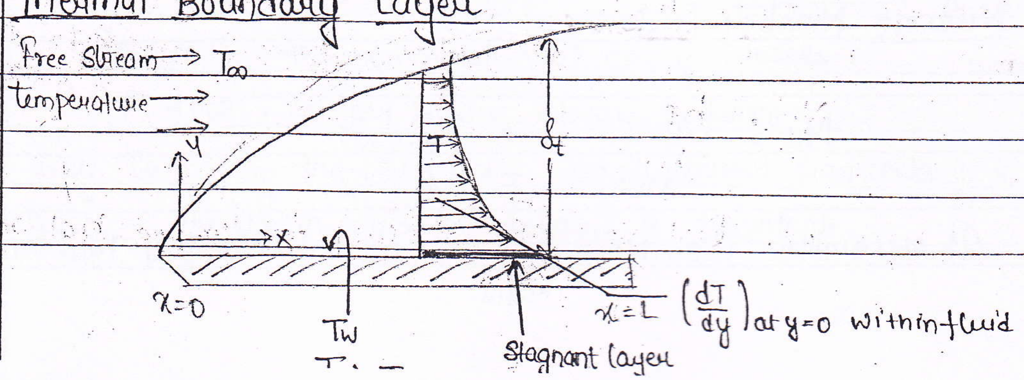
Pr No signifies the relative magnitudes of momentum and heat diffusion rates through the fluid i.e. which is diffusing faster.

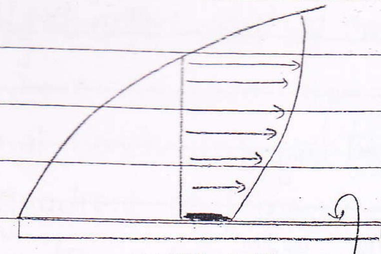
Pr of air = 0.69 to 0.72

Pr of Water = 2 to 6

Pr of Hg → Very low since $k_{Hg}$ is very high.

Thermal Boundary Layer






Just similar to hydrodynamic boundary layer inside which velocity gradient are present normal to plate, thermal boundary layer is also a thin layer inside which temp. gradients are present in the normal dirxn to plate

✓ $\boxed{\text{heat transfer rate is a vector quantity.}}$

These temp. gradients arise due to transfer of heat from the hot plate to the fluid. i.e. if momentum diffusion causes velocity ~~distribu~~ gradients in H.B.L, heat diffusion causes temp. gradient within the T.B.L.

✓ Conduction occurs normal to plate (in y-dirxn) and Convection in (x-dirxn)

The mode of heat transfer from the hot plate to cold fluid to the stagnant layer of fluid at $y=0$ is by conduction only. Once heat get conducted through this layer it is simultaneously conducted in y-dirxn through the fluid layers and also convected by moving layers of fluid.

Consider a small differential area $dA$ on the plate at a distance '$x$' from the leading edge through which the differential heat transfer rate b/w plate and fluid is $dq$,

∴ from fourier law of conduction and newton's law of cooling,

Under steady state, $dq = -k_f\, dA \left(\frac{\partial T}{\partial y}\right)_{\substack{\text{at } y=0 \\ \text{within fluid}}} = h_x\, dA (T_w - T_\infty)$

($h_x$ = Local Convective H.T. Coefficient)

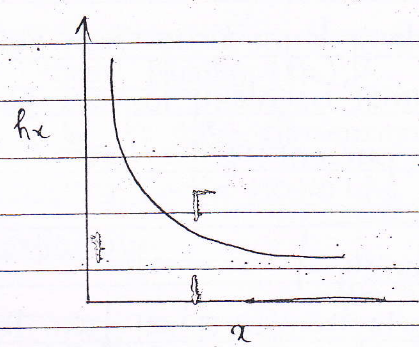
$\frac{\partial T}{\partial y}$ ← spatial gradient of temp. within the fluid

$$\Rightarrow h_x = \frac{-K_f \left(\frac{\partial T}{\partial y}\right)_{at\, y=0}}{(T_w - T_\infty)}$$

As $x$, increases, $h_x$ decreases due to $\left(\frac{\partial T}{\partial y}\right)_{at\, y=0}$ decreases

⇓

क्योंकि fluid गरम होने लगा है.




A.T.E-2009 A Coolant fluid at 30°c flows over a heated flat plate maintained at a constant temp. of 100°c, the boundary layer temp distribution at a given location on a plate may be approximated as $T = 30 + 70 \exp(-y)$ where $y$ is in metre measured in dirxⁿ normal to plate and T°c, if $K_{fluid} = 1 W/m^2\text{-}K$, The local Convective heat transfer Coeff. $h_x (W/m^2K)$ in that location will be

(a) 0.2 (b) 1 (c) 5 (d) 10

Ans. $T = 30 + 70(\exp(-y))$

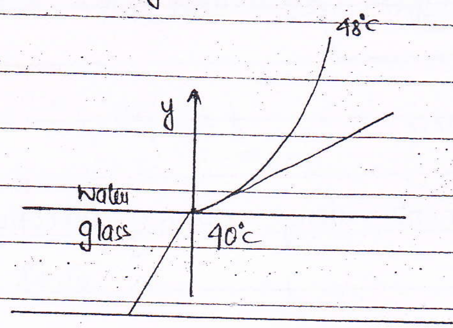
$$\Rightarrow \frac{\partial T}{\partial y} = -70 e^{-y}$$

at $y = 0$

$$\Rightarrow \frac{\partial T}{\partial y} = -70$$

$$\Rightarrow h_x = \frac{-1 \times (-70)}{(100 - 30)} = 1 W/m^2K$$

G.A.T.E-2003 Heat is being transferred by Convection from Water at 48°c to a glass plate whose surface that is exposed to the Water is at 40°c. The thermal Conductivity of Water = 0.6 W/mK, Kglass = 1.2 W/mK. the spatial gradient of temp. in the Water at the Water glass interface is $\frac{dT}{dy} = 1\times10^4$ K/m.




(i). the value of temp. gradient in the glass in Water glass interface in K/m is

(a) $-2\times10^4$ (b) (0.0) (c) $0.5\times10^4$ (d) $2\times10^4$.

(ii). The heat transfer Coefficient h in $W/m^2K$ is

(a) 0.0 (b) 4.8 (c) 6, ~~(d) 750~~ (Water has 'h' always > 300)

Ans. ~~Thermal resistance at Water glass Interface is~~

$K_f = 0.6$ W/mK

$$dq = +\underset{(water)}{K_f}\, dA \left(\frac{dT}{dy}\right)_{\substack{at\ y=0 \\ within\ water}} \leftarrow \text{as T is increasing with y.}$$

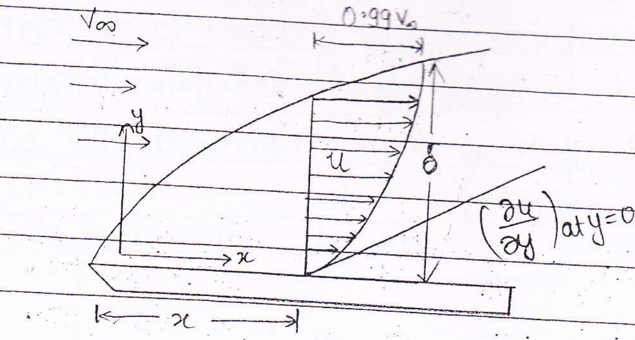
$$= h\, dA\,(T_{water} - T_{glass}) \quad (\text{as Under Steady state}).$$

$$= +K_{glass}\, dA \left(\frac{dT}{dy}\right)_{at\ y=0\ within\ glass}$$

On equating, $h = 750\ W/m^2K$

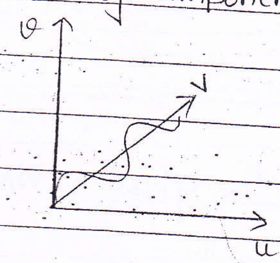
and $\left(\frac{dT}{dy}\right)_{y=0\ within\ glass} = 0.5\times10^4$.

# Momentum Eqⁿ for H.B.L




$u = f(x, y)$

$u, v$ are velocity components along $x$ and $y$ directions.




Considering only inertia and viscous forces i.e. neglecting pressure forces,

$$\overset{\oplus}{\sum} F_x = m a_x$$

We get momentum eqⁿ as

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$ [Navier stoke's eqⁿ of motion along x-dirxⁿ]

↑ K.V of fluid

By solving differential eqⁿ with boundary conditions,

At any $x$, $u = 0$ at $y = 0$

$u = 0.99 V_\infty$ at $y = \delta$

$\frac{\partial u}{\partial y} = 0$ at $y = \delta$

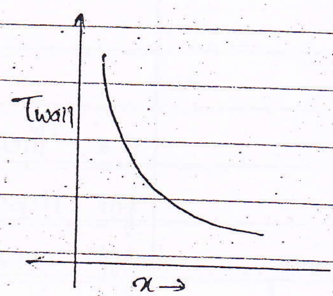
$\frac{\partial^2 u}{\partial y^2} = 0$ at $y = 0$ [as $\frac{\partial u}{\partial y}$ is max. at $y = 0$]

We get velocity distribution,

$$\boxed{\frac{u}{V_\infty} = \frac{3}{2}\left(\frac{y}{\delta}\right) - \frac{1}{2}\left(\frac{y}{\delta}\right)^3}$$

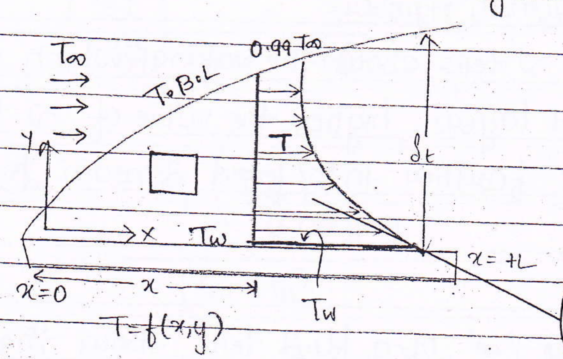
$\tau_{wall}$ = Local wall shear stress at any $x$,

$$= \mu\left(\frac{\partial u}{\partial y}\right)_{at\, y=0} = \mu V_\infty \left(\frac{3}{2\delta}\right)$$




as $\delta \propto \sqrt{x} \Rightarrow \tau \propto \frac{1}{\sqrt{x}}$

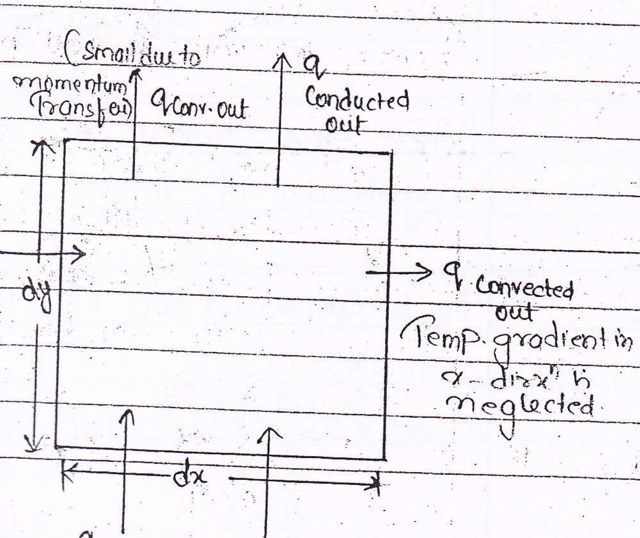
## Energy eq$^n$ for Thermal boundary layer




$\left(\frac{dT}{dy}\right)_{at\, y=0}$ = spatial gradient of temperature within the fluid at $y=0$.

Assume

- Steady State Conditions.
- $x$-dir$x^n$ Conduction neglected.



Energy don't have dirxⁿ.

for steady state conditions,

Total energy entering into element = Total energy leaving from element.

$$\Rightarrow \quad u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

** There is a striking similarity b/w the momentum eqⁿ given for H·B·L in terms of u (velocity distribution) and the energy eqⁿ given for T·B·L in terms of terms of temperature.
The solⁿ to both the eqⁿs could exactly be the same when the kinematic viscosity of fluid is equal to its thermal diffusivity i.e. when $P_r = 1$. ( for most Engineering fluids $P_r \approx 1$).

Significance of Prandtl number
Kinematic viscosity '$\nu$' tells about the rate at which momentum diffusion occurs ~~to~~ through the fluid layers, higher the value of $\nu$ the viscous influence is felt farther into free stream hence the value of

H·B·L ← $\delta$ will be more.

Thermal diffusivity '$\alpha$' of a fluid tells about the rate at which heat diffuses through the fluid, higher the value of $\alpha$, it means temp. influence of plate is felt farther into the free stream. hence ~~AT wall~~ $\delta_t$ will be more.

Hence, the Prandtl no, ratio b/w $\nu$ and $\alpha$ signifies the relative magnitudes of thickness of H·B·L and thickness of T·B·L

$P_r$ is connecting link b/w H·B·L and T·B·L

| | | |
|---|---|---|
| If $P_r > 1$ | If $P_r < 1$ | If $P_r \simeq 1$ |
| $\Rightarrow \nu > \alpha$ | $\Rightarrow \nu < \alpha$ | $\Rightarrow \nu \approx \alpha$ |
| $\delta > \delta_t$ | $\Rightarrow \delta < \delta_t$ | $\Rightarrow \delta \approx \delta_t$ |
| $\frac{\delta_t}{\delta} < 1$ | $\Rightarrow \frac{\delta_t}{\delta} > 1$ | $\Rightarrow \frac{\delta_t}{\delta} \approx 1$ |

$$\boxed{\frac{\delta_t}{\delta} = \frac{1}{1.026}(P_r)^{-1/3}}$$

At $y=0 \Rightarrow T = T_w$

At $y=\delta_t \Rightarrow T = 0.99\,T_\infty$

At $y=\delta_t \Rightarrow \frac{\partial T}{\partial y} = 0$

$\frac{\partial^2 T}{\partial y^2} = 0$ at $y=0$

The temperature distribution within ToBoL is

$$\frac{T - T_w}{T_\infty - T_w} = \frac{3}{2}\left(\frac{y}{\delta_t}\right) - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

const.

$\frac{\partial^2 T}{\partial y^2}$ ~~= 0. at y ≠ 0~~

$$\Rightarrow \left(\frac{\partial T}{\partial y}\right)_{at\,y=0} = \frac{3}{2\delta_t} \times (T_\infty - T_w)$$

$$h_x = -k_f \frac{\left(\frac{\partial T}{\partial y}\right)_{at\,y=0}}{(T_\infty - T_w)}$$

$$\frac{\partial T}{\partial y\,(T_\infty - T_w)} = \frac{3}{2}\left(\frac{1}{\delta_t}\right) - \frac{3}{2}\left(\frac{y^2}{\delta_t^3}\right)$$

$$\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{3}{2} \times \left(\frac{1}{\delta_t}\right) \times (T_\infty - T_w)$$

$$\Rightarrow h_x = +k_f \times \frac{3}{2} \times \frac{1}{\delta_t}$$

$\delta_t = f(\delta, P_r)$

But $\delta = f(Re_x)$

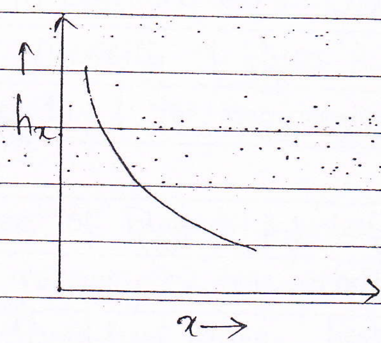
$\Rightarrow \delta_t = f(Re_x, P_r) \Rightarrow h_x = f(x, k_f, Re_x, P_r)$

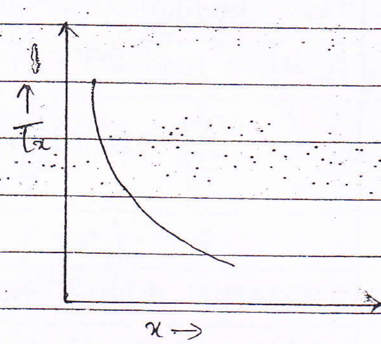
hence, the local Convective heat transfer Coefficient in forced Convection over flat plates is given by

(local Nusselt no.) $Nu_x = \dfrac{h_x x}{K} = 0.332\, Re_x^{1/2} \cdot Pr^{1/3}$

$$\Rightarrow \frac{h_x x}{K} = 0.332 \left(\frac{V_\infty x \rho}{\mu}\right)^{1/2} \left(\frac{\mu C_p}{K}\right)^{1/3}$$

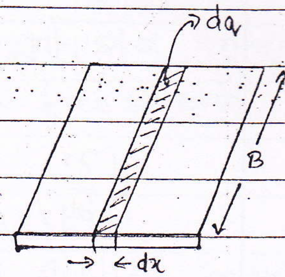
$$\Rightarrow h_x \propto x^{-1/2} \Rightarrow h_x \propto \frac{1}{\sqrt{x}}$$




There is Similarity b/w $h_x$ and $T_x$.

Average Convective H.T. Coefficient $= \bar{h} = \dfrac{1}{L}\displaystyle\int_0^L h_x\, dx$




$$dq = h_x B\, dx\, (T_w - T_\infty)$$

$$\Rightarrow \int dq = Q = \int_0^L h_x B\, dx\, (T_w - T_\infty)$$

$$= \bar{h} \times (BL) \times (T_w - T_\infty)$$

$$\bar{h} = 2 \times (h_{x=L})$$

i.e. Twice the local heat Convective H.T Coefficient at Trailing edge.

Average Nusselt No = $\bar{Nu} = \frac{\bar{h}L}{k} = 0.664\ Re_L^{1/2}\ Pr^{1/3}$

Where $Re_L$ is local Reynold's no. at Trailing edge $Re_L = \left(\frac{V_\infty L \rho}{\mu}\right)$

$\tau_{avg}$ = Average Wall Shear stress = 2 × Local shear stress at trailing edge.

Comparison, a partial similarity.

Reynolds- Colburn Analogy (Analogy b/w. H.T. & fluid friction)

$$\tau_\omega = \frac{\rho V_\infty^2}{2}(C_{fx})$$

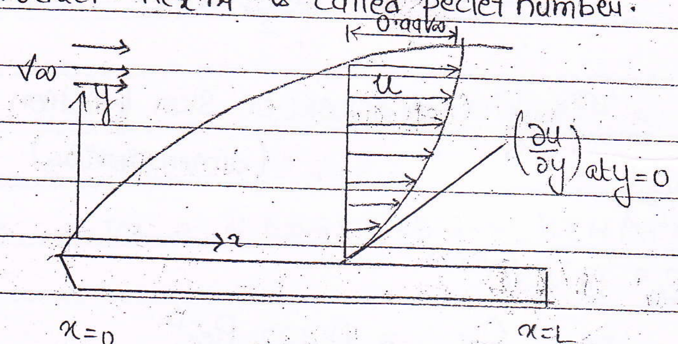
$C_{fx}$ = Drag Coeff. or skin friction Coeff.

The significance of this analogy is that we would be able to predict the value of local ~~heat~~ convective heat transfer Coeff. ($h_x$) at same location of plate by knowing $C_{fx}$ at the same location even when there is no heat transfer b/w the plate and the fluid.

Stanton No. $St_x = \frac{Nu_x}{Re_x\ Pr}$

The product $Re_x\ Pr$ is called peclet number.

7/12/2011




Local wall shear stress at any $x$,

$$\tau_\omega = \mu\left(\frac{\partial u}{\partial y}\right)_{at\,y=0} = \mu V_\infty \frac{3}{2\delta}$$

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But $\delta = \frac{4.64x}{\sqrt{Re_x}}$

$$h_x = \frac{-k_f\left(\frac{\partial T}{\partial y}\right)_{at\,y=0}}{T_w - T_\infty}$$

$$\frac{T - T_w}{T_\infty - T_w} = \frac{3y}{2\delta_t} - \frac{1}{2}\left(\frac{y}{\delta_t}\right)^3$$

$$\left(\frac{\partial T}{\partial y}\right)_{at\,y=0} = \frac{3}{2\delta_t}(T_\infty - T_w)$$

$$h_x = \frac{3}{2\delta_t} \times k_f$$

$$Nu_x = 0.332\ Re_x^{1/2} \cdot Pr^{1/3} = \frac{h_x x}{k}$$

$\Rightarrow$ Local Wall shear stress $= \mu V_\infty \frac{3}{2\delta} = \mu V_\infty \frac{3}{\frac{2 \times 4.64x}{\sqrt{Re_x}}}$ —— ②

Where $R_x = \frac{V_\infty x \rho}{\mu} = \frac{V_\infty x}{\nu}$ {local Reynold's no.}

$\frac{3}{2}\frac{\rho V_\infty^2 x \mu}{4.64x(\rho V_\infty x)/\sqrt{Re_x}}$

Local Wall shear stress,

$$\tau_w = \frac{\rho V_\infty^2}{2} \times C_{fx} \text{ —— ①}$$

$\Rightarrow \frac{3}{2}\frac{\rho V_\infty^2}{4.64} \frac{x\sqrt{Re_x}}{Re_x}$

$C_{fx}$ = Drag Coefficient or skin friction coefficient (dimensionless)

$\Rightarrow 0.332\ \rho V_\infty^2 \times (Re_x^{-1/2})$

Equating $Eq^n$ ① & ②

$$\Rightarrow \frac{C_{fx}}{2} = 0.332\ Re_x^{-1/2}$$

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$$\frac{Nu_x}{Re_x Pr} = 0.332 \frac{Re_x^{1/2} \cdot Pr^{1/3}}{Re_x \cdot Pr} = \text{Stanton No. } (St_x) = \frac{h_x}{\rho V_\infty C_p}$$

$$\Rightarrow \frac{Nu_x}{Re_x Pr} = 0.332\, Re_x^{-1/2} Pr^{-2/3}$$

$$\Rightarrow \boxed{St_x \cdot Pr^{2/3} = \frac{C_{fx}}{2}}$$

~~Significance of this analogy is that~~

I.E.S-1993 Air ~~flows through~~ over a heated flat plate at a velocity of 50 m/s, the local skin friction Coefficient at a point on the plate is 0.004, estimate the local heat transfer Coefficient at this point. The property data for air are given as

$\rho_{air} = 0.88\,kg/m^3$, Viscosity $= 2.286\times10^{-5}\,kg/m\,sec$.

$C_p = 1.001\,KJ/kg\text{-}K$, $K = 0.035\,\frac{W}{mK}$

Ans.

$$Pr = \frac{\mu C_p}{K} = \frac{2.286\times10^{-5}\times1.001\times10^3}{0.035} = 0.653$$

$$St_x \times (0.653)^{2/3} = \frac{0.004}{2} = 0.0124$$

$$\Rightarrow St_x = \cancel{0.0124}\ 2.655\times10^{-3}$$

$$\Rightarrow \frac{h_x}{\rho V_\infty C_p} = \cancel{0.0124}\ 2.655\times10^{-3}$$

$$\Rightarrow h_x = 2.655\times10^{-3}\times0.88\times50\times1001 = 117\,W/m^2K$$

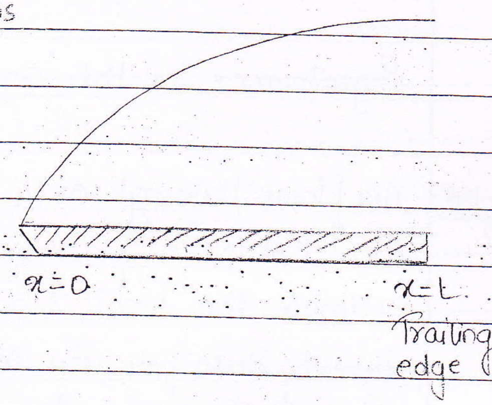
Prair always in the range of
0.69, 0.7

S-2001

Air at 25°c flows over a thin plate with a velocity of 2.5 m/s, the plate is 2m long and 1m wide, estimate the thermal boundary layer thickness at the trailing edge of the plate and total drag force experienced by the plate, at 25°c for air,

$\rho = 1.2\ kg/m^3$, $\nu = 15 \times 10^{-6}\ \frac{m^2}{sec}$, $P_r = 0.69$

Ans. Need not be that H.B.L & T.B.L are always formed.
if temp. of fluid in that of plate then no T.B.L but always we get H.B.L when fluid flows




$$\delta = \frac{4.64x}{\sqrt{Re_x}}$$

$$Re_x = \frac{V_\infty x}{\nu}$$

$$= \frac{2.5 \times 2}{15 \times 10^{-6}}$$

$$= \frac{10^6}{3}$$

$$\Rightarrow \delta = \frac{4.64 \times 2 \times \sqrt{3}}{10^3} = 16.07 \times 10^{-3} m$$

$$\Rightarrow \cancel{\frac{\delta}{\delta_t}} \quad \frac{\delta_t}{\delta} = \frac{1}{1.026} P_r^{-1/3}$$

$$P_r = \frac{\mu c_p}{K} = 0.69 \text{ given}$$

$$\Rightarrow \frac{\delta_t}{16.07 \times 10^{-3}} = \frac{1}{1.026} \times (0.69)^{-1/3} = 15.66 \times 10^{-3} m$$

$$\delta_t = 15.66 \times 10^{-3} m$$

$$\cancel{\tau_w} = \frac{C_{fx}}{2} = \cancel{0.332 \left(\frac{10^6}{3}\right)^{1/2}} \quad 0.332 \times \left(\frac{3}{10^6}\right)^{1/2}$$

$$= \cancel{181.68 \times 2} = \cancel{383.36} = 1.15 \times 10^{-3}$$

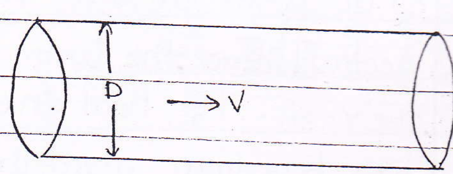
$$\text{Drag force} = \tau_{avg} \times \text{Area}$$
$$= 2 \times \left( \tau_{at\ \cancel{tail}\ x=L} \right) \times \text{Area}$$

$$(\tau_w)_{at\ x=L} = \frac{3\mu V_\infty}{\frac{2 \times 4.64 \times 2}{\sqrt{Re_x}}} = \frac{3}{16.07} \times 10^{3} \times 2.5 \times 1.2 \times 15 \times 10^{-6}$$
$$= \frac{8.437 \times 10^{-3}}{2} \times \cancel{2}$$

$$\Rightarrow \text{Drag force} = \cancel{0.016875 \times 2} \quad 8.437 \times 10^{-3}$$

Forced Convection in flow through pipes

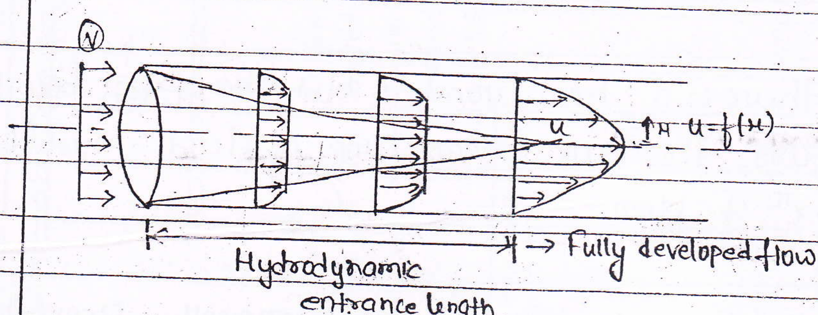
$$Re = \frac{VD\rho}{\mu}$$




If $Re < 2000 \Rightarrow$ Laminar flow
If $Re > 4000 \Rightarrow$ Turbulent flow

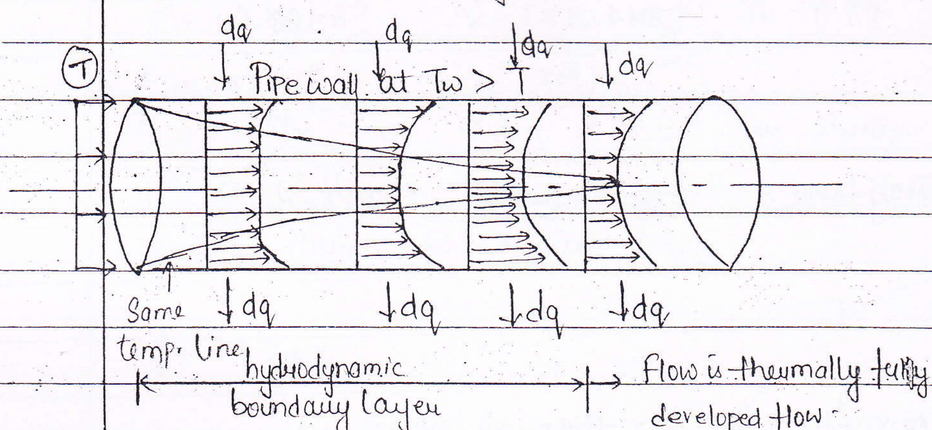
~~Hydrodynamic interaction and Thermal interaction~~

Hydrodynamic entrance length




Hydrodynamic entrance length is the distance travelled by fluid from entrance of the pipe upto the section where the viscosity has totally penetrated from pipe boundary upto the axis of pipe beyond which flow is said to be fully developed flow. It is lesser for laminar flow.

Thermal entrance length




T.E.L is the distance travelled by the fluid from the fluid upto the section where the temp. influence of the hot pipe is felt throughout the x-s/c. i.e. heat energy has diffused completely from the pipe boundary upto the axis of the pipe.

Bulk mean temp. ($T_b$) of a fluid at a given x-s/c is the average temp which takes into account the variation of 'T' w.r.t radius measured from the axis and hence indicates the total thermal energy transported by the fluid at that section given by

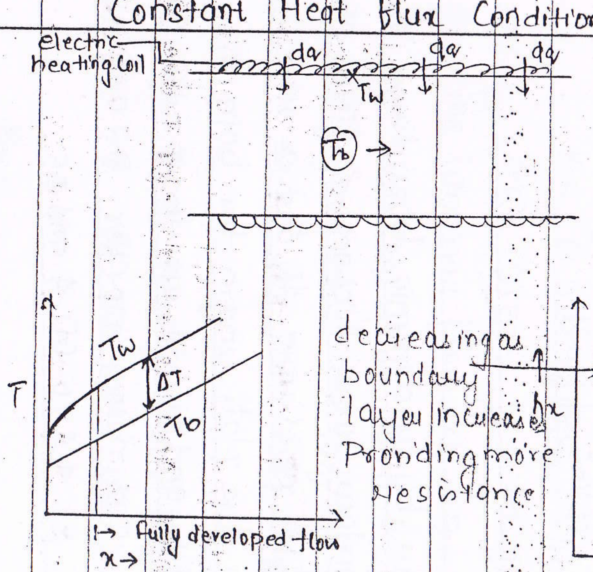
$$\dot{m} c_p T_b = \rho \times \frac{\pi}{4} \times D^2 \times U_{mean}$$

Whenever there is a heat transfer b/w the pipe boundary and the fluid, the bulk mean temp. of fluid must change in the dirxn of flow.

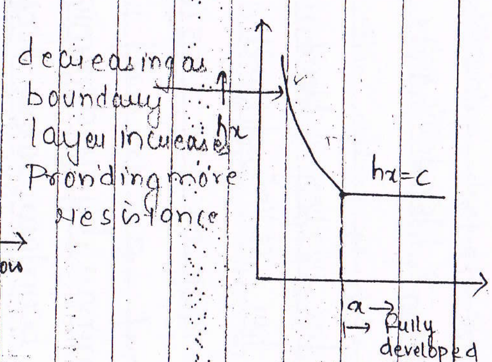
There are 2 different situations that generally prevails during forced convection in flow through pipes.

- Constant heat-flux Condition.
- Constant Wall temp. Condition

## Constant Heat flux Condition

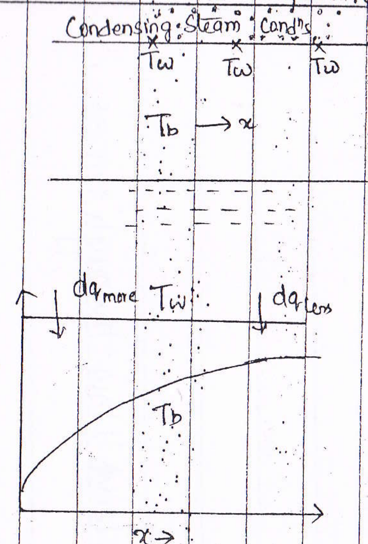



Heat flux $= h_x dA\, \Delta T\ W/m^2$




Valid for both the cases

## Constant wall temperature




** Constant heat flux Conditions can be maintained at the boundary of the pipe by supplying electric current in the coil wound over the pipe. Whereas Constant wall temp Conditions can be maintained at the surface of the pipe by condensing steam or vapour around the pipe. In both these cases the bulk mean temp. of the fluid must increase in the dir^n of flow since it is receiving heat.

** Since the local convective heat transfer coeff. is remaining constant for fully developed flow, the wall temp. of the pipe must increase in the dir^n of flow if constant heat flux cond^ns are maintained, also in such case ΔT remains constant in the dir^n of flow where ΔT is equal to difference b/w wall temp. and bulk mean temp.

In constant wall temp - conditions, since $h_x$ is constant in dir^n of flow and ΔT↓ in that dir^n, the local heat flux dq must decrease in that dir^n of flow.

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** It is not possible to maintain both the above Conditions at the same time for a given pipe.

For fully developed laminar flow with constant heat flux conditions

$$Nu = C = \frac{48}{11} = \frac{hD}{K}$$

for fully developed laminar flow with constant wall temp conditions

$$Nu = 3.66$$

For fully developed Turbulent flow through pipes.

$$Re > 4000$$

$$Nu = 0.023\ Re^{0.8} Pr^{n} \rightarrow \text{Mc adam's eq}^{n}.$$

Where,

$n = 0.4$ for heating of fluid

$n = 0.3$ for cooling of fluid

A.T.E-2011 The ratios of the laminar hydrodynamic boundary layer thickness to thermal boundary layer thickness of flows of two fluids P and Q on a flat plate are 1/2 and 2 respectively. The Reynold's no. based on the plate length for both the flows is $10^4$, the Prandtl and nusselt no. for P are 1/8 and 35 resp. The prandtl and nusselt no. for Q are resp.

(a). 8 and 140 (b) 8 & 70 (c) 4 & 70 (d) 4 and 35

G.A.T.E-2005 An uninsulated air Conditioning duct of rectangular x-s 1m x 0.5m Carrying air at 120°c with a velocity of 10m/s is exposed to an ambient of 30°c, neglect the effect of duct Construction material for air in the range of 20-30°c, Data are as follows.

$K = 0.025\ W/mK$ . viscosity $= 18\mu$ Pa-sec.

$P_r = 0.73$ $\rho = 1.2\ kg/m^3$

(A) The laminar flow $Nu = 3.6$ for Constant wall temp. Cond^ns and for turbulent flow $Nu = 0.023\ Re^{0.8}\ P_r^{0.33}$

(i) The Reynolds no. for the flow is:

(a) 444 (b) 890 (c) $4.44 \times 10^5$ (d) $5.33 \times 10^5$

(ii) The heat transfer rate per metre duct in Watts is

(a). 3.8 (b) 53 (c) 89 (d) 769

Ans. Part :-

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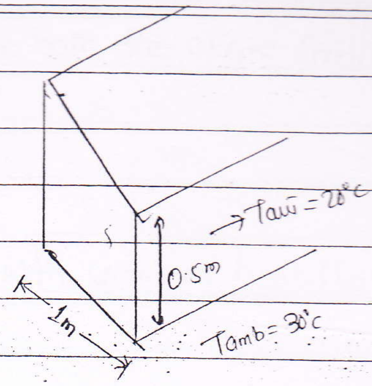
Characteristic dimension

$$D = \frac{4A_{c/s}}{P}$$

$$= \frac{4(1\times0.5)}{2[1+0.5]}$$

$$= 0.667m$$




$$Re = \frac{VDe}{\mu} = \frac{0.667\times1.2\times10}{18\times10^{-6}} = 4.44\times10^{5}$$ (hence the flow is turbulent Since Re > 4000)

$$\frac{hD}{k} = Nu = 0.023(4.44\times10^{5})^{0.8}\times(0.73)^{.33}$$

$$\Rightarrow h = 25.6\ W/m^2K$$

$$dq = 25.6\times \cancel{\pi}\times 2\times(1+0.5)\times1\times10$$
$$= 768$$

G.A.T.E-1999 : If velocity of water inside a smooth tube is doubled, the turbulent flow heat transfer Coeff. b/w the water and the tube will (a).

(a). Remain Unchanged
(b). Increase to double its value
(c). Increase but will not reach double its value.
(d). Increase to more than double its value.

Ans. In turbulent flow,

$$h \propto V^{0.8}$$

$$\Rightarrow \frac{h_2}{h_1} = \cancel{(0.2)}\ (2)^{0.8} = 1.74$$

I.E.S-1997 Air flows through a 25 mm diameter tube with a mean velocity of 30 m/s. the tube wall temp. is 280°c and the air temp. increases from 20°c to 260°c. Using the simple reynold's analogy calculate the length of the tube required and the pumping power. for a turbulent flow in a tube $f = \frac{0.046}{Re_d^{0.2}}$

The properties of air may be taken as

$K = 38.45 \times 10^{-3}\ W/m°c$

$c_p = 1.0268\ KJ/kg°c$

$\rho = 0.7306\ kg/m^3$, $\mu = 26.17 \times 10^{-6}\ kg/m\,sec.$

Ans. Reynold's analogy for turbulent flow through pipes:-

$$\text{Stanton No} = \frac{Nu}{Re\,Pr} = \frac{Nu}{\text{Peclet no}} = \frac{h}{\rho v c_p}$$

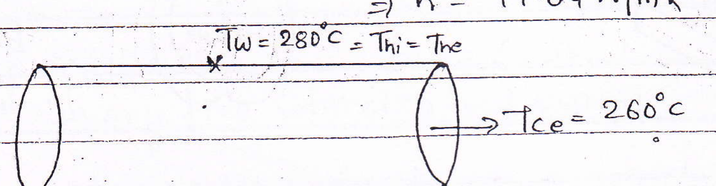
$$Re = \frac{VD\rho}{\mu} = 2.0938 \times 10^4 \ (\text{Turbulent}) \quad Re > 4000 \qquad \boxed{D = \frac{4 \times A_{x\text{-sec}}}{P}}$$

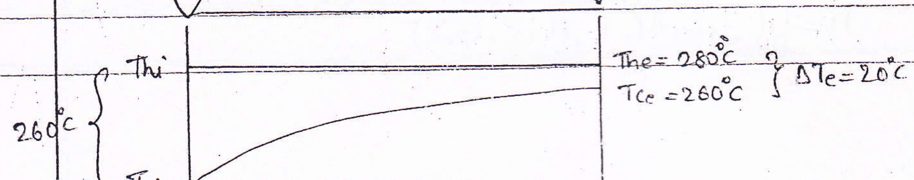
$$\text{friction factor } f = \frac{0.046}{Re_d^{0.2}} = \frac{0.046}{7.31} = 6.28 \times 10^{-3}$$

$$St = \frac{Nu}{Re\,Pr} = \frac{h}{\rho v c_p}$$

$$\boxed{St\ No = \frac{f}{8}} = 7.86 \times 10^{-4} = \frac{h}{\rho v c_p}$$

$$\Rightarrow h = 17.69\ W/m^2K$$



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$$\dot{m} = \rho \times \frac{\pi}{4} D^2 \times \vec{V}$$

$$= 0.7306 \times \frac{\pi}{4} \times (0.025)^2 \times 30$$

$$= 0.0107 \text{ kg/s}$$

$$\Delta T_m = LMTD = \frac{\Delta T_i - \Delta T_e}{\ln\left(\frac{\Delta T_i}{\Delta T_e}\right)} = 93.57$$

$$\Rightarrow \quad 17.69 \times (\pi \times 0.025 \times L) \times 93.57 = 0.0107 \times C_{p\,air} \times (T_{ce} - T_{ci})$$

$$= 0.0107 \times 1.0268 \times 1000 \times 240$$

$$\Rightarrow \quad L = \underline{\underline{20.2\,m}}$$

Pumping Power $= \omega \times Q \times h_f$

$\omega \rightarrow$ specific weight

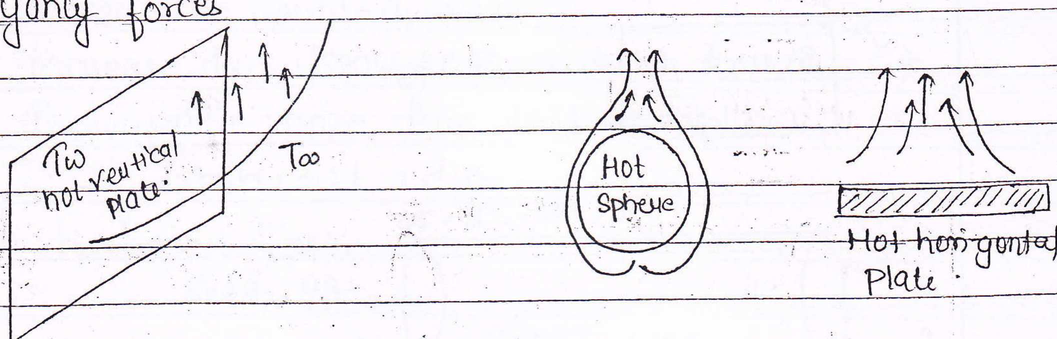
$Q \rightarrow$ discharge

$h_f \rightarrow$ head loss due to friction.

$$h_f = \frac{f l v^2}{2gD}, \quad \omega = \rho g.$$

<u>Free Convection heat transfer</u> (Natural Convection)

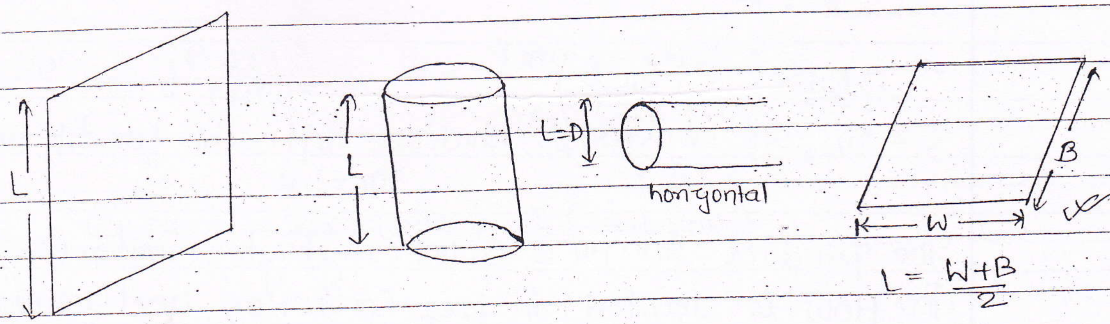
Flow occurs naturally due to density changes, and the resulting buoyancy forces




$$h = f(g, \beta, \Delta T, L, \mu, \rho, c_p, k)$$

L → Characteristic length of body.

For Vertical Walls & Cylinders




$L = \frac{W+B}{2}$

$\beta$ = Isobaric Volume expansion Coefficient

$$= \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_{P=C}$$

$\beta$ signifies the ability of fluid to undergo greater density changes for a given temp. rise.

more $\beta$ more buoyancy, more $h$, (how quickly density is changing ~~with t~~)

from dimensional analysis all the variables can be grouped into 3 dimensionless numbers.

- Grashoff Number (Plays same role in free Convection as that of Re in forced Convection)
- Nusselt No (Nu)
- Prandtl No (Pr)

$$\text{Grashoff No } (Gr) = \frac{\text{Inertia force} \times \text{Buoyancy force}}{(\text{Viscous force})^2} = \frac{g\beta\Delta T L^3}{\left(\frac{\mu}{\rho}\right)^2}$$

$\therefore$ In any free convection heat transfer, $Nu = f(Gr, Pr)$

$\frac{hD}{k} = f(Gr, Pr)$

Usually in any free Convection heat transfer

$$Nu = C\,(Gr\,Pr)^m$$

C & m are Constants which are different for different Cases.

The product Gr · Pr is Called rayleigh number (Ra).
The flow is decided in free Convection heat transfer
Whether laminar or turbulent based on value of Gr · Pr
product.

If $Gr\,Pr > 10^9$ ⇒ Flow is turbulent $(m = 1/3)$

If $Gr\,Pr < 10^9$ ⇒ flow is laminar $(m = 1/4)$

$$\beta = \frac{1}{T_{mean}(K)}$$

Mean $T_{mean}$ = Mean film Temperature $= \left(\frac{T_w + T_\infty}{2}\right)$

$Nu = \frac{hL}{k}$ ⇒ $h = \boxed{\;}$ Ans.

I.E.S - 2009 Estimate the Coefficient of heat transfer from a Vertical plate 2m×2m to the surrounding air at 25°c. The plate surface temp is 150°c. Also calculate the rate of heat transfer from the plate. For air assume $\nu = 1.6 \times 10^{-5}\ m^2/sec$. The properties of air at film temp. are

$\rho = 0.972\ kg/m^3$ $\quad Pr = 0.69$

$C_p = 1.009\ kJ/kg\text{-}K$, $\quad k = 3.13 \times 10^{-2}\ W/mK$

the Constants C and m in the nusselt no. eqⁿ are 0·15 and $\frac{1}{3}$ resp

Ans. as m is given $\frac{1}{3}$ ⇒ flow is turbulent.

$$Gr = \frac{g\beta \Delta T L^3}{(\mu/\rho)^2} = \frac{9.81 \times \frac{1}{\cancel{298}\,(298+150+273)} \times 125 \times 2}{(1.6\times10^{-5})^2}$$

$$= 3.21 \times 10^{10}$$

$$Gr.Pr = \cancel{2.2\times10^{10}}\ \cancel{9.1\times10^{9}} > 10^9 \text{ (~~turbulent~~)}$$

$$= 1.83\times10^{10} > 10^9$$

$$\Rightarrow Nu = 0.15\,(1.83\times10^{10})^{1/3}$$

$$= 395.5 = \frac{h\times 2}{3.13\times10^{-2}} \qquad L = \frac{2+2}{2} = 2$$

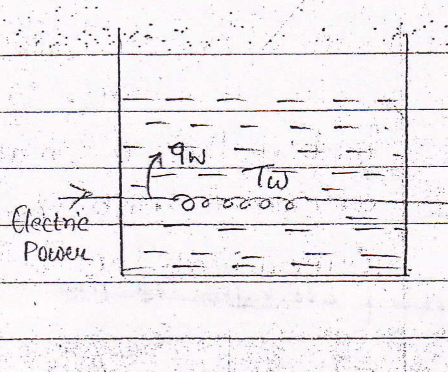
$$\frac{4A}{P} = 2$$

$$\Rightarrow h = \cancel{6.19}\ 9.6\ W/m^2K$$

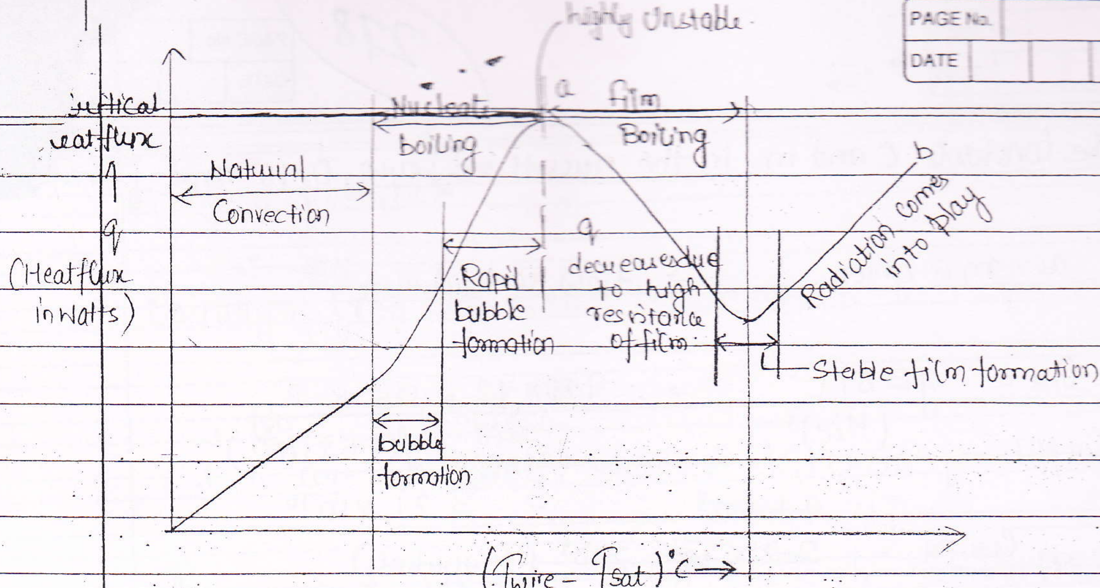
$$q = 9.6\times(2\times2)\times(150-25) = \underline{4.9\,kW}$$

## Pool Boiling Curve

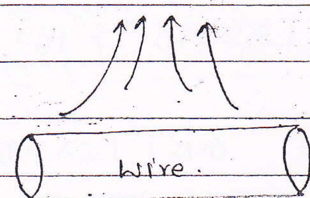
Boiling of liquid occurs whenever the liquid comes into Contact with a solid surface whose temp is greater than saturation temp. of liquid.



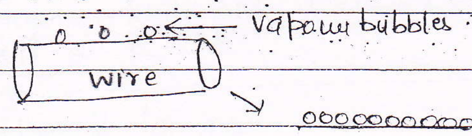

$T_w$ = Temp. of wire
$T_s$ = Sat. temp. of liquid.




o Natural Convection

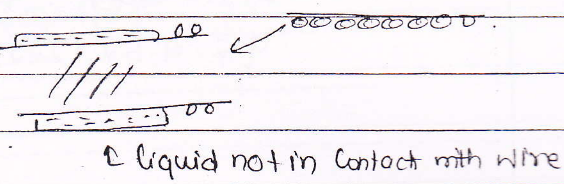



o Nucleate Boiling




o film Boiling

The film is bad Conductor Of heat ~~transfer~~.




** Heating of the wire or supplying electric power must be stopped before reaching state 'a' because beyond state 'a' the system is in highly Unstable eqlb^m and eqlb^m is established on at 'b', but the wire may result in its melting due to high temp associated with b. The heat flux supplied at state 'a' is called Critical heat flux.